AFGL-TR-77-0203
INSTRUMENTATION PAPERS, NO. 258

Computer Programs for Three-Dimensional Cable Problems in Tethered-Balloon Applications

JOHN B. WRIGHT

15 September 1977

Approved for public release; distribution unlimited.

AEROSPACE INSTRUMENTATION DIVISION PROJECT 6665
AIR FORCE GEOPHYSICS LABORATORY
HANSCOM AFB, MASSACHUSETTS 01731

AIR FORCE SYSTEMS COMMAND, USAF

This report has been reviewed by the ESD Information Office (OI) and is releasable to the National Technical Information Service (NTIS).

This technical report has been reviewed and is approved for publication.

FOR THE COMMANDER

John Howard

Chief Scientist

Qualified requestors may obtain additional copies from the Defense Documentation Center. All others should apply to the National Technical Information Service.

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER: AFGL-TR-77-0203, AFGL-IP-258
2. GOVT ACCESSION NO.:
3. RECIPIENT'S CATALOG NUMBER:
4. TITLE (and Subtitle): COMPUTER PROGRAMS FOR THREE-DIMENSIONAL CABLE PROBLEMS IN TETHERED-BALLOON APPLICATIONS
5. TYPE OF REPORT & PERIOD COVERED: Scientific. Interim.
6. PERFORMING ORG. REPORT NUMBER: IP No. 258
7. AUTHOR(s): John B. Wright
8. CONTRACT OR GRANT NUMBER(s):
9. PERFORMING ORGANIZATION NAME AND ADDRESS: Air Force Geophysics Laboratory (LCB), Hanscom AFB, Massachusetts 01731
10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS: 62101F, 66650902
11. CONTROLLING OFFICE NAME AND ADDRESS: Air Force Geophysics Laboratory (LCB), Hanscom AFB, Massachusetts 01731
12. REPORT DATE: 15 September 1977
13. NUMBER OF PAGES: 191
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office):
15. SECURITY CLASS. (of this report): Unclassified
15a. DECLASSIFICATION/DOWNGRADING SCHEDULE:
16. DISTRIBUTION STATEMENT (of this Report): Approved for public release, distribution unlimited.
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report):
18. SUPPLEMENTARY NOTES:
19. KEY WORDS (Continue on reverse side if necessary and identify by block number):

Tethered-balloons
High-altitude tethered balloons
Computer programs
Calculator programs
Hewlett-Packard
Cable forces
Aerodynamic forces
Three-dimensional solutions
Wind loads on cables

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Three computer/calculator programs were developed to solve tethered-balloon cable problems. The programs provide information on cable tension, geometry, and spacial positions from the balloon to the surface with a variable profile of wind speeds and azimuths. One program allows the holding of a constant cable length and the determination of a new balloon altitude and cable profile for any number of different wind fields. Both printed and plotted outputs are provided. Complete documentation including flow-charts is provided to permit the programs to be adapted to computers having different language than the next page

DD FORM 1473, 1 JAN 73 — EDITION OF 1 NOV 65 IS OBSOLETE

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

409 578

20. (Cont)

Hewlett Packard 9810 used in their development.

Preface

The author wishes to recognize the contributions of Gregory A. Vayda, 2nd Lt., USAFR, who contributed the plotter routine used in one of the computer programs. In addition, Mr. Robert Vespirini, Emmanual College, was helpful with many of the details and reviews of the contents of these programs as well as those in Reference 1.

Preceding Page Blank - Not Filmed

Contents

Illustrations

Tables

PRECEDING PAGE BLANK - NOT FILMED

Computer Programs for Three-Dimensional Cable Problems in Tethered-Balloon Applications

1. INTRODUCTION

In Reference 1, six computer programs for use in the solution of various tethered-balloon problems were developed and documented therein. Program No. 76.006 in Reference 1 provides a means of obtaining the tension, elevation angle, and space-position of a single tether cable of any size and weight from the balloon to the ground for any balloon altitude and any two-dimensional wind profile. The wind profile from the ground up to the balloon may include winds of any magnitude but all must lie in the same azimuth plane, however, they can have ± signs. It was therefore desirable that a three-dimensional case be developed for use in a similar type calculator/computer to permit a completely realistic entry of atmospheric wind conditions and a three-dimensional evaluation of the cable geometry and other physical parameters.

Program Nos. 77.007 and 77.007P presented herein, are three-dimensional programs. They retain many of the features of Program No. 76.006 including the option of specifying a cable (cylinder) drag coefficient or calculation of a variable drag coefficient based on local Reynolds Numbers at various altitudes. Entry of wind magnitude and azimuth at up to twelve different altitudes is permitted. The programs are longer than No. 76.006 chiefly due to the complexities of a three-

(Received for publication 15 September 1977)

1. Wright, John B. (1976) Computer Programs for Tethered-Balloon System Design and Performance Evaluation, AFGL-TR-76-0195.

dimensional solution, the addition of a plotting routine, and the extra "housekeeping" required in dealing with azimuth values. Program Nos. 77.007 and 77.007P differ by (a) the number of optional ways to make repetitive computational runs and (b) the plotting of parameters in 77.007P. A third program, No. 77.007B, allows one to retain a cable length computed with 77.007, change the winds, and then determine a new balloon altitude and cable geometry.

2. EQUIPMENT

The program, in three versions, documented herein was developed using a Hewlett-Packard Model 9810A calculator/computer. All versions require nearly full use of the 2036 program steps (HP Option 003), 111 storage registers (HP Option 001), the paper tape printer (HP Option 004), and the MATH ROM (HP No. 11210A). Additional items required are:

Program No. 77.007	Printer-Alpha ROM (HP No. 11211A),
Program No. 77.007B	Printer-Alpha ROM (HP No. 11211A),
Program No. 77.007P	Printer-Plotter ROM (HP No. 11261A) and an HP Model 9862A Plotter.

As with those in Reference 1, these programs were written with the idea that they could be adopted to other less capable machines and they do not necessarily make full use of potentials of the 9810A.

Table 1 defines the symbols used in the program listings. Unlike Reference 1, the listings (Sections 3.3.6, 3.4.6, and 3.5.6) are not direct copies of the output tape listing where many mnemonics can have two meanings depending on operational mode. Instead the listings here are program forms showing meaningful mnemonics both in and out of the alpha numeric mode. In addition, the mathematics shown in display register columns x, y, and z on the forms will aid in understanding the operation of the programs.

Table 1. Program Codes

A. Standard Mode — Computation

In the definitions below, x, y, and z represent the contents of display registers x, y, and z respectively; a and b represent the contents of memory registers a and b respectively. The mnemonics and their respective functions are shown below.

Mnemonic	Function
π	$\pi \to x$
b	$b \to x$
a	$a \to x$
$y\to$	$y \to$ memory register which follows
$x\to$	$x \to$ memory register which follows
$1/x$	$1/x \to x$
IND	Used for indirect addressing
x()	Puts the value in the following memory address into x
x^2	$x^2 \to x$
R↑	$x \to y,\ y \to z,\ z \to x$
↓	$z \to y,\ y \to x,\ z \to z$
$x \rightleftarrows y$	$y \to x,\ x \to y$
↑	$x \to y,\ y \to z,\ x \to x$
$\sqrt{x}$	$\sqrt{x} \to x$
÷	$y/x \to y$
X	$xy \to y$
−	$y - x \to y$
+	$y + x \to y$
CHG S	$-x \to x$
ENT E	Used to assign an exponent to a number being entered into x
CLR	Set to 0, x, y, z, a, and b
0 through 9	0 through $9 \to x$
.	Used to place a decimal point in a number being entered into x
CNT	(Continue) Used as a null operation in a program. Used in running of a program.
LABEL	Used in conjunction with a following symbol to indicate a position in the program memory.

PNT	Prints the value in x. When multiple PNT's are used, lines are skipped after x is printed.
$x < y$	If $x < y$, branch to address indicated by number in next 4 steps; if not, skip the next four steps.
$x = y$	If $x = y$, branch to address indicated by number in next 4 steps; if not, skip the next four steps.
$x > y$	If $x > y$, branch to address indicated by number in next 4 steps; if not, skip the next four steps.
Go To	Go to the memory location specified in the next steps.
END	Used as the last step in a program; Set point of operation to Step 0000.
STOP	Causes program to stop and permit entries.
x^y	$x^y \rightarrow x$
ln	Natural logarithm of $x \rightarrow x$
e^x	$e^x \rightarrow x$
arc	Used in conjunction with sin, cos and tan keys to obtain inverse trigonometric functions.
sin x	$\sin x \rightarrow x$
cos x	$\cos x \rightarrow x$
tan x	$\tan x \rightarrow x$
TAB , 4	Common logarithm of $x \rightarrow x$
TAB , 9	Rounds number in y to the power of 10 indicated by integer value of number in x. Rounded number $\rightarrow$ x, y unchanged.

B. Alpha Numeric Mode — Printing

The alph ι numeric mode is entered by using FMT step twice and is exited by using FMT once.

A through Z	A through Z respectively is printed
0 through 9	0 through 9 respectively is printed
#	# is printed
÷ or /	/ is printed
CLR	This causes a carriage return or move to next line to be printed.
CNT	This causes a blank space in printing.

C. Plotter Commands

The following combinations of key strokes serve to operate the plotter.

FMT, ↑	Lifts pen
FMT, ↓	Drops pen
FMT, 1 , ↑	Lifts pen, scales coordinates, moves to coordinates.
FMT, 1 , ↓	Scales coordinates, moves to coordinates, drops pen.
FMT, 1 , 1	Symbol scale factor from x
FMT, 1 , 2	Scales X-coordinate from x and y
FMT, 1 , 3	Scales Y-coordinate from x and y
FMT, 1 , 4	Drops pen, draws + at point plotted.
FMT, 1 , 5	Draws X-axis.
FMT, 1 , 6	Draws Y-axis
FMT, 1 , FMT	Initiates plotter alpha mode
FMT	Terminates plotter alpha mode.

D. Special Commands

FMT, Go To	Automatically loads program card(s)
FMT, x→	Records data in storage onto card(s)
FMT, x()	Automatically loads data card(s)
To POLar	Converts rectangular coordinates in x and y registers to polar coord. with result placing angle in y and radius in x.

3. PROGRAMS FOR TETHERED-BALLOON CABLE, THREE-DIMENSIONAL CASE, VARIABLE WIND PROFILE, OPTIONAL INTERNAL VARIABLE DRAG COEFFICIENT

3.1 General Description

Typical concerns in the design of a tethered-balloon flight system are the ability: (a) to lift the weight of the cable; (b) to maintain the cable tension below its working limit; (c) to retain a reasonable cable tension at the winch without the cable laying on the ground; and (d) to keep the balloon within an acceptable area overhead under widely varying wind conditions.

All of the buoyancy and aerodynamic forces introduced into a cable system by the balloon can be summed and defined by a single force and its angle. This total force, F_T, and the angle, θ, can be computed by use of either Program Nos. 76.003, 76.004, or 76.005 in Reference 1. These two parameters are then treated as inputs into either the two-dimensional cable Program No. 76.006, Reference 1, or these three-dimensional programs.

The basic forces acting on the cable, in addition to the total balloon force acting at the top of the cable, are the cable weight, the aerodynamic drag, and the resultant restraining force at the ground winch. The weight of the cable per thousand feet is specified while the force at the winch is part of the problem solution.

The drag of the cable is complex since it is a variable function of the atmospheric density, wind velocity, and cable diameter. Since the programs were intended for use with balloon altitudes of up to 66,000 ft MSL, the effect of Reynolds Number on drag coefficient could not be ignored. Reynolds Number is directly proportional to cable diameter, atmospheric density, and wind speed, and inversely proportional to atmospheric coefficient of viscosity. It was further assumed that the cable cross-section would be circular so that, in effect, the cable can be considered to be a cylinder—or a series of cylinders. Accordingly, the program was designed to permit the user to specify either a fixed cylinder (cable) C_D held constant throughout the altitude range or a program computing C_D which varies with altitude, wind velocity, cable diameter, Reynolds Number, etc.

In concept, the cable is broken into rigid elements of a specified length, K, and the forces acting on this length evaluated to a net magnitude and angle with which the next lower element must align and provide equal restraint, Figure 1A. Thereby a series of outputs is provided at each of these many points proceeding downward from the balloon to and including the surface. Some of these outputs are the cable tension, space-position, elevation and azimuth angles, and length.

Because of the three-dimensional capability of the programs, not only must the wind velocity be calculated at each of the elements but its direction must similarly be evaluated. A table of altitude-wind speed-wind azimuth, part of the initial user entries, is utilized by making straight-line interpolations for each of the two wind parameters at each of the element altitudes. (See Section 3.2.9.)

During the downward progression of calculations, the cable tension and pitch angle are monitored for the condition of zero tension or horizontal cable. Under such conditions, the balloon has not provided sufficient lifting force for the size and weight of the cable and the cable is said to be unable to reach the ground. For a given balloon, cable, and atmospheric condition, a lower flight altitude is therefore suggested. If such an event occurs just at the surface, the cable is lying on the ground. Hauling in some cable would bring the balloon to a lower altitude and lift the cable off the ground.

When the cable reaches down to the surface where the winch would be located—or where the tension becomes zero or the cable horizontal—sufficient details are presented as a final output to allow construction of the three-dimensional geometry of the cable as well as the compass azimuths of several components.

Table 2 is useful as an aid in selecting which of the three programs is best used for a particular problem. Programs 77.007 and 77.007P, nearly identical programs except for ending rerun options and a plotting routine in the latter, are based on solving a cable program where the balloon altitude is fixed at a specified level. Both will give identical answers to a given problem. Both provide the option of restarting the same program or of lowering the balloon to other fixed altitudes specified by the table of winds which was originally entered.

Program 77.007 offers two additional options; (a) a rerun with a different cable without having to re-enter a table of winds, and (b) the ability to hold the cable length just calculated constant, change the winds and find where the balloon may reach an equilibrium altitude. This latter option requires use of Program No. 77.007B. While making use of the same mathematics as the other two programs, its logic and details are somewhat different. The two programs, 77.007 and 77.007B, are designed to be used together and therefore each can be used to call in the other.

Program No. 77.007P makes the following plots on a single piece of paper:

(a) Altitude vs H (Y) Displacement,

(b) Altitude vs I (X) Displacement,

(c) H vs I (X vs Y),

(d) Altitude vs Tension,

(e) Altitude vs Cable Elev. Angle,

(f) Altitude vs Effective Dynamic Pressure.

As with all programs so far introduced in this series, these three programs treat the static condition and do not attempt to consider the dynamics of balloon or cable motion.

Table 2. Highlights of Programs

Program No. 77.007	Program No. 77.007P
Requires 2 Cards—4 Sides	Requires 2 Cards—4 Sides

Plus 2 Cards—3 Sides
Data Cards with Density
and C_D Constants for
either Program

User Entries: Balloon altitude, Surface altitude, Cable C_D or Internal Computation of C_D, Cable diameter, Cable weight, Element length to be used in solution, Balloon total force and angle, Winds from balloon to surface—magnitude and direction at up to twelve altitudes

Program Solves for and Prints: Cable Tension and Angle along cable and at the winch, Cable Length, Space Positions of points along the cable and Angles of the elements, Relative positions of the balloon and winch, Sighting Angles and Slant Range of the balloon from the winch.

Program No. 77.007	Program No. 77.007P
No Plotting	Plots 6 Curves—Cable Tension, Cable Elevation Angle, Effective Dynamic Pressure, X, and Y vs Altitude and X vs Y.
Rerun Options after each Problem Solution 0 - New Prob. Start over, all entries required. 1 - New Prob. but only cable parameters, balloon total force, and angle entries—Alt./Winds held from previous problem. 2 - Repeat runs with same cable with balloon automatically lowered to each altitude in original wind profile. Bln force and angle entered at each lower altitude. 3 - Hold cable length found in orig. Opt. 0 or 1 runs, change winds—? New balloon altitude. Requires Prog. No. 77.007B. Auto. Read-In Call for 77.007B Cards when this Opt. specified.	Rerun Options after each Problem Solution 0 - New Prob. Start over, all entries required. 2 - Repeat runs with same cable with balloon automatically lowered to each altitude in original wind profile. Bln force and angle entered at each lower altitude.

Program No. 77.007B

Requires 2 Cards—4 Sides
Data Cards not required

User Entries: Estimated Balloon total force and angle at an estimated alt. to be found by program and the new wind profile.

Program Solves for and Prints: New balloon altitude, Wind magnitude and direction at that altitude, plus same parameters as 77.007 for this new condition

No Plotting

Rerun Options after each Problem Solution
0 - New Prob. Start over. Requires Prog. No. 77.007. Auto. Read-In Call for 77.007 Cards when this Opt. specified.
3 - Continue to hold cable length, Change winds again.

3.2 Development of Equations and Programs

The basic formulae and description hereunder apply to all three programs. However, there are some details which apply to 77.007 and 77.007P or are at least better understood by following the logic in these programs. Any differences in approach with 77.007B are explained in Section 3.4.1.

These tether-cable programs will handle the three-dimensional case where the azimuth and velocity of the wind may vary with altitude or over the whole cable length. Thus the balloon, cable, and winch will not necessarily be located in one vertical plane. The wind velocity and direction at up to 12 altitude points will be made a part of the input data required. Straight-line interpolations of both velocity and azimuth angle will be made part of the program computations for all intermediate altitudes. A wind vector will be calculated for each incremental cable element by determining the altitude of the bottom-end point of each element, determining the velocity and azimuth of the wind at that altitude, and assuming that these conditions are constant over the complete element length.

The objective of the computation is to (starting at the top of the tether-cable where it is attached to a balloon's confluence-point with a known balloon force vector) evaluate the cable tension, elevation and azimuth angles, space position, etc., moving downward to the earth's surface where the cable terminates onto a winch. The cable can be considered to be made up of a series of short and rigid cylindrical elements of length, K, attached by freely pivoting connectors, Figure 1A. Each element will lie at an angle in line with the tension vector solved for the element immediately above it.

3.2.1 TENSION VECTOR

The first cable element below the balloon, Figure 1A, is contained in the vertical plane defined by the wind vector at the balloon since it also contains the balloon total force, F_T, at an elevation angle, θ. The wind vector V_1, acting on the first element will not lie in the same vertical plane and is shown (looking down from the balloon) rotated clockwise from the balloon wind vector and initial vertical plane by an angle, α. Solution of a free-body diagram using only the V_C component of the wind, the element weight, and the tension, F_T, would provide a solution whereby the next element below would lie in the same initial vertical plane. That is essentially how the two-dimensional case—Program No. 76.006—is handled with a single vertical plane containing all elements of the cable from the balloon to the ground winch.

However, when the side component of the wind, V_S, is included, the resulting side force (drag) will rotate the bottom-end tension vector out of the initial vertical plane. A change in elevation angle will also occur as in the two-dimensional case.

Figure 1A is expanded in Figure 1B to illustrate all of the forces, angles, and linear dimensions. It also indicates how the total wind vector, V_1, must be broken into components in order to obtain the total aerodynamic drag and then resolve it in turn into three manageable components. The wind vector can first be divided into two components; one normal to the cable V_N, which will be considered the total drag producer, and one parallel to the cable, V_A, which will be assumed to produce negligible skin-friction drag.

The value of V_N will be used to calculate Reynolds Number in order to select the drag coefficient from a stored table of cylinder C_D – R for the option in which the program calculates a drag coefficient for each element. The C_D, whether computed in this fashion or entered as a constant, is then used with atmospheric density and V_N^2 to calculate the total drag, D_T, of the element.

The total drag can be divided into three components:

(a) D_W in the vertical direction,

(b) D_H one of the two horizontal components lying in the vertical plane containing the cable element,

(c) D_S the other horizontal component perpendicular to the aforementioned vertical plane.

The angle, ϕ, is both the angle of the wind vector, V_N, above the horizontal and the angle of the total drag, D_T, below the horizontal.

Equations can therefore be developed as follows—the numerical subscripts are deleted at this point for clarity:

Given as known quantities: V, θ, and α:

$$V_C = V \cos \alpha \tag{1}$$

$$V_A = V_C \cos \theta = V \cos \alpha \cos \theta \tag{2}$$

$$V_N^2 = V^2 - V_A^2 = V^2 - V^2 \cos^2 \alpha \cos^2 \theta \tag{3}$$

$$V_N^2 = V^2 (1 - \cos^2 \alpha \cos^2 \theta) \tag{4}$$

$$V_N = V\sqrt{(1 - \cos^2 \alpha \cos^2 \theta)} = V \cos \phi \tag{5}$$

$$\cos \phi = \sqrt{1 - \cos^2 \alpha \cos^2 \theta} \tag{6}$$

$$\sin \phi = V_A/V = \frac{V \cos \alpha \cos \theta}{V} = \cos \alpha \cos \theta\ . \tag{7}$$

Considering the drag vectors:

$$\sin \phi = D_W/D_T \longrightarrow D_W = D_T \sin \phi \tag{8}$$

Therefore

$$D_W = D_T \cos\alpha \cos\theta \tag{9}$$

$$\cos\phi = D_D/D_T \rightarrow D_D = D_T \cos\phi \tag{10}$$

$$\cos\alpha = D_H/D_D \rightarrow D_H = D_D \cos\alpha . \tag{11}$$

Therefore

$$D_H = D_T \cos\phi \cos\alpha \tag{12}$$

$$\sin\alpha = D_S/D_D \rightarrow D_S = D_D \sin\alpha . \tag{13}$$

Therefore

$$D_S = D_T \cos\phi \sin\alpha . \tag{14}$$

Eq. (5) is used to obtain velocity for the Reynolds Number/C_D extraction and in the solution for total drag in:

$$D_T = C_D \frac{1}{2} \rho V_N^2 A \qquad (A = \text{Diam in ft} \times K) \tag{15}$$

Eqs. (9), (12), and (14) are then used to resolve D_T into three components that may be introduced into the free-body diagram as follows:
Summing the forces (aerodynamic, weight, tension):

(a) Horizontal Direction in Vertical Cable—Element Plane,

$$F_T \cos\theta + D_H = T_1 \cos\theta_1 \cos\beta_1 \tag{16}$$

(b) Horizontal Direction Perpendicular to the Vertical Cable—Element Plane,

$$D_S = T_1 \cos\theta_1 \sin\beta_1 \tag{17}$$

(c) Vertical Direction,

$$F_T \sin\theta = W + D_W + T_1 \sin\theta_1 . \tag{18}$$

Eqs. (16), (17), and (18) contain three unknowns: T_1, θ_1, and β_1.

From Eq. (17)

$$T_1 = \frac{D_S}{\cos\theta_1 \sin\beta_1}. \tag{19}$$

Substituting in Eq. (16)

$$F_T \cos\theta + D_H = \frac{D_S}{\cos\theta_1 \sin\beta_1} \cos\theta_1 \cos\beta_1$$

$$F_T \cos\theta + D_H = \frac{D_S}{\tan\beta_1} \tag{20}$$

$$\beta_1 = \text{arc tan} \frac{D_S}{F_T \cos\theta + D_H} \tag{21}$$

From Eq. (16)

$$T_1 = \frac{F_T \cos\theta + D_H}{\cos\theta_1 \cos\beta_1} \tag{22}$$

Substituting in Eq. (18)

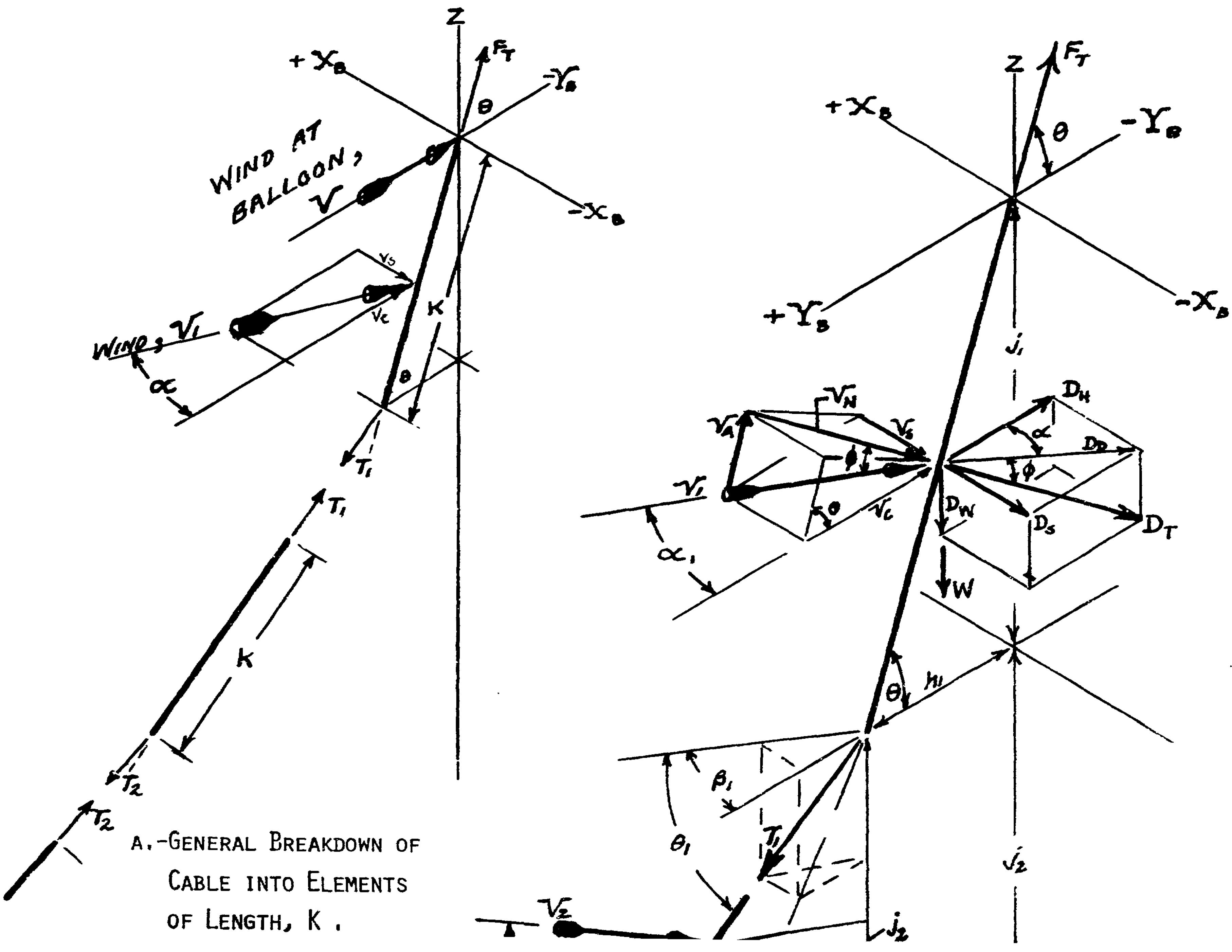
$$F_T \sin\theta - W - D_W = \frac{F_T \cos\theta + D_H}{\cos\theta_1 \cos\beta_1} \sin\theta_1$$

$$= \frac{F_T \cos\theta + D_H}{\cos\beta_1} \tan\theta_1$$

$$\theta_1 = \text{arc tan} \frac{F_T \sin\theta - W - D_W}{F_T \cos\theta + D_H} \cos\beta_1 \tag{23}$$

$$T_1 = \frac{F_T \cos\theta + D_H}{\cos\theta_1 \cos\beta_1}. \tag{24}$$

Eqs. (21), (23), and (24) are utilized for the solution defining the tension vector at the bottom end of the cable element with which the next element aligns. In subsequent loops through these equations a value of T_i at the top of an element would be used instead of F_T used for the first element. The particular forms of equations developed were chosen over other possible forms to avoid indeterminate solutions. For example, when $\beta = 0$-deg, the $\sin\beta = 0$ and the tension would be infinity if Eq. (19) was incorporated into the program. It is assumed that β will never equal 90° particularly if small K values are selected.



A.-GENERAL BREAKDOWN OF CABLE INTO ELEMENTS OF LENGTH, K.

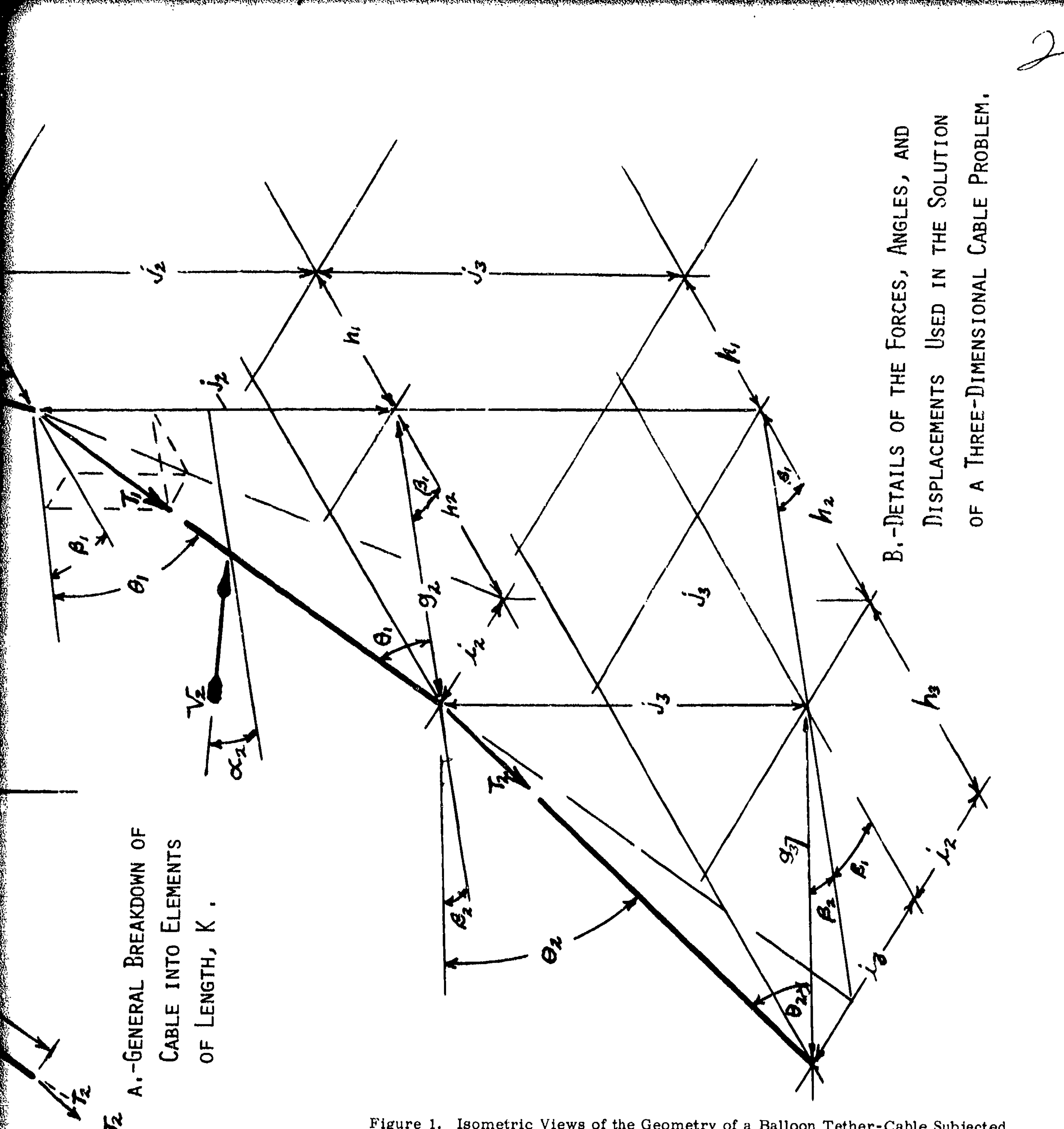


Figure 1. Isometric Views of the Geometry of a Balloon Tether-Cable Subjected to Winds of Varied Magnitudes and Azimuth Angles

A set of axes is defined as shown in Figure 1B which will be the reference axes during the main body of computations. Since the origin is at the top of the cable at the confluence point of the balloon, they are annotated as follows: (1) Y_B axis positive ahead of balloon in the direction from which the balloon wind is blowing, and (2) X_B axis positive 90° clockwise from the Y_B axis—looking down from above the balloon. The reason for this particular set of axes will be apparent in subsequent discussion.

As evident in Figure 1B, the initial element lies in the Y_B-Z vertical plane. The solution of β_1, θ_1, and T_1 not only establish the conditions at the top end-point of the next element below but also define a new vertical plane containing this second element. This second plane is rotated by the angle β_1 clockwise from the Y_B-Z plane for the example shown; the figure basically shows the positive sign conventions for angles and distances. This second vertical plane containing the second element becomes the plane to which its wind vector, V_2, is referenced to determine the associated relative wind angle, α_2. A repeat of all the above computations will then provide solutions for β_2, θ_2, and T_2 and therefore conditions defining the third element down.

While the angle β will usually be small, the angle α can have any magnitude. In these programs, a value up to 360° is permitted since the triginometric output of most calculators will handle a full 360°. In the example shown in Figure 1B, α lies between 0° and 90°. In this case the vertical component of the total drag is downward, D_H in a direction to cause a decrease in the cable elevation angle, and D_S in a direction to cause a clockwise rotation of the cable. These are all positive in the sense that they were used as illustrated when writing the free-body equations. As the angle α is increased to 90° and beyond, the three drag components change directions and at times some disappear as illustrated in Figure 2. However, as can be checked by substitution of appropriate trig functions in Eqs. (9), (12), and (14), proper values and signs result for any value of α as used in these equations.

3.2.2 CABLE GEOMETRY

Returning to Figure 1B, it is apparent that while determining the progression of cable tension and angles working down the cable, that the linear movement of the cable must be calculated and both angular and linear data must be properly summed as computations proceed. As stated previously, the first cable element below the balloon lies in the vertical Y_B-Z plane. The position of the bottom end-point of that element is defined by:

$$j_1 = K \sin \theta \tag{25}$$

where j_1 is the vertical drop

$$h_1 = K \cos \theta \tag{26}$$

where h_1 is the horizontal displacement from the top of the element in the Y direction.

However, the next elements move out of the Y_B-Z plane and as the cable responds to side loads, the bottom end point of each element is displaced in both the X and Y directions. Similarly, the vertical plane containing the cable element moves to a different azimuth than the next element above. Knowledge of this azimuth is required with the next wind azimuth in order to define the relative wind angle, α, acting on the element.

$$j_2 = K \sin \theta_1 \ . \tag{27}$$

$$g_2 = K \cos \theta_1 \ . \tag{28}$$

$$h_2 = g_2 \cos \beta_1 = K \cos \theta_1 \cos \beta_1 \ . \tag{29}$$

$$i_2 = g_2 \sin \beta_1 = K \cos \theta_1 \sin \beta_1 \ . \tag{30}$$

At the next or third element, further complications arise since the above solutions would provide displacements along and perpendicular to the g_2 direction. Figure 3, which is an X – Y projection of Figure 1B, will clarify the following: While

$$h_3' = K \cos \theta_2 \cos \beta_2 \tag{31}$$

$$h_3 = K \cos \theta_2 \cos (\beta_2 + \beta_1) \tag{32}$$

or

$$h_i = K \cos \theta_{i-1} \cos (\beta_{i-1} + \beta_{i-2} + \dots + \beta_1) \tag{33}$$

and

$$H = h_i + h_{i-1} + h_{i-2} + \dots h_1 = \Sigma h \ . \tag{34}$$

While

$$i_3' = K \cos \theta_2 \sin \beta_2 \tag{35}$$

$$i_3 = K \cos \theta_2 \sin (\beta_2 + \beta_1) \tag{36}$$

or

$$i_i = K \cos \theta_{i-1} \sin (\beta_{i-1} + \beta_{i-2} + \dots + \beta_1) \tag{37}$$

and

$$I = i_i + i_{i-1} + i_{i-2} + \dots i_2 = \Sigma i \tag{38}$$

$$j_3 = K \sin \theta_2 \tag{39}$$

or

$$j_i = K \sin \theta_{i-1} \tag{40}$$

$$J = j_i + j_{i-1} + j_{i-2} + \dots + j_1 = \Sigma j \, . \tag{41}$$

Eqs. (33), (37), and (40) are thus used to determine the displacement of the bottom end-point of one element from that of the element immediately above. Eqs. (34), (38), and (41) provide the summation of these distances to the balloon which become part of both the element output and the final output when the surface is reached.

The sum of the β angles, as used in Eqs. (33) and (37), when added to the azimuth of the wind at the balloon provides the azimuth of the vertical plane containing the cable element. The relative wind angle, α, to which the element is being subjected must be determined. A positive convention is established whereby α is always positive with values between 0° and 360°. By definition:

$$\alpha = \text{Wind AZ} - \text{Element AZ} \, .$$

As in Program No. 76.006, wind (and density) conditions are found for the altitude of the bottom end-point of the element but are assumed to exist as a constant over the whole element length, K. Therefore:

$$\alpha_1 = \text{Wind AZ at: } (Z_B - j_1) - \text{Wind AZ at: } Z_B \tag{42}$$

$$\alpha_2 = \text{Wind AZ at: } [Z_B - (j_1 + j_2)] - \text{Wind AZ at: } Z_B - \beta_1 \tag{43}$$

$$\alpha_3 = \text{Wind AZ at: } [Z_B - (j_1 + j_2 + j_3)] - \text{Wind AZ at: } Z_B - (\beta_2 + \beta_1) \tag{44}$$

or

$$\alpha_i = \text{Wind AZ at: } (Z_B - J_i) - \text{Wind AZ at: } Z_B - (\beta_{i-1} + \beta_{i-2} + \dots + \beta_1) \, . \tag{45}$$

Expressed in shorthand symbols used in some figures, this becomes

$$\alpha_i = AZ_i - (\beta_B + \Sigma\beta) \, . \tag{46}$$

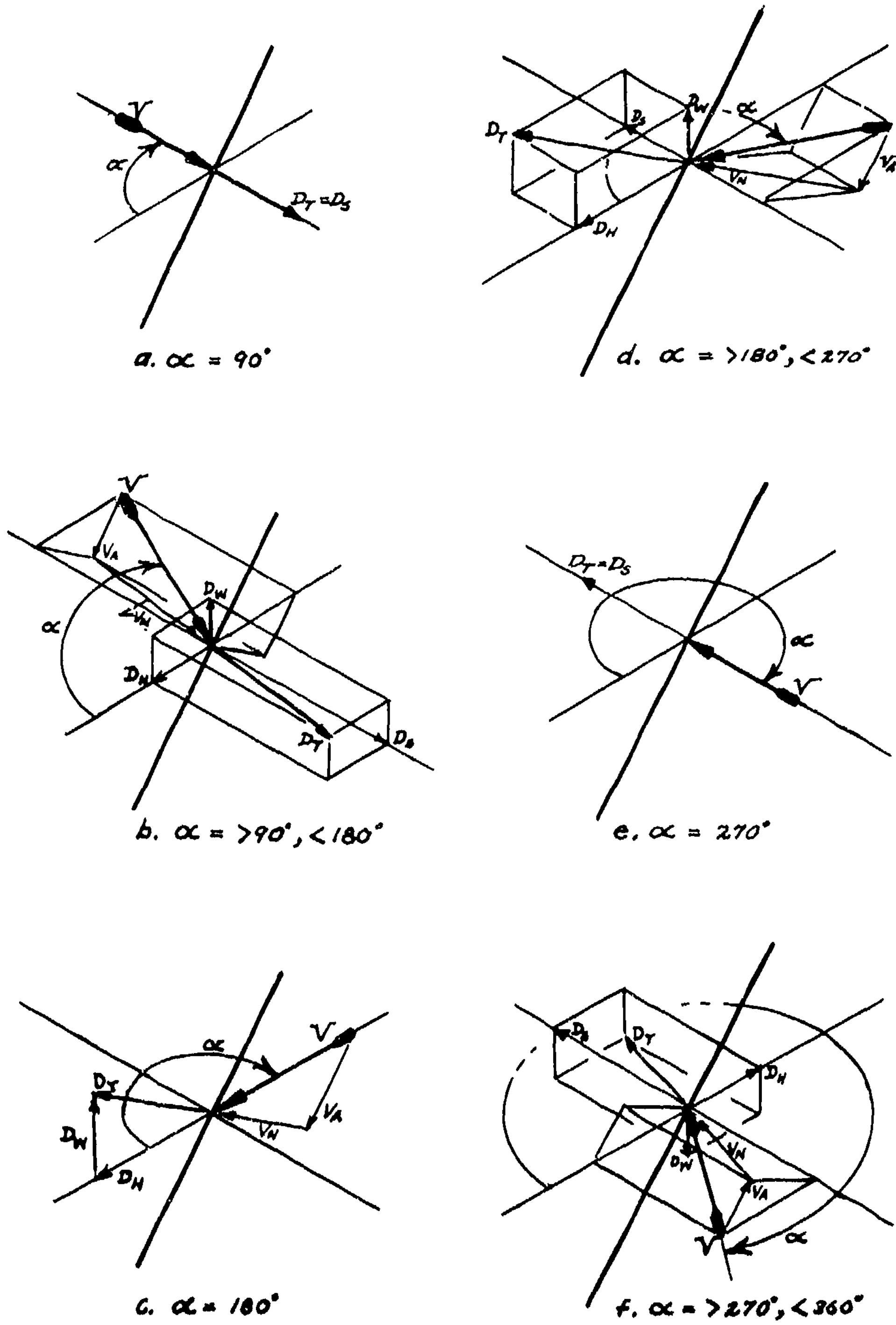


Figure 2. Resolution of Drag Components for Various Magnitudes of the Relative Wind Angle, α

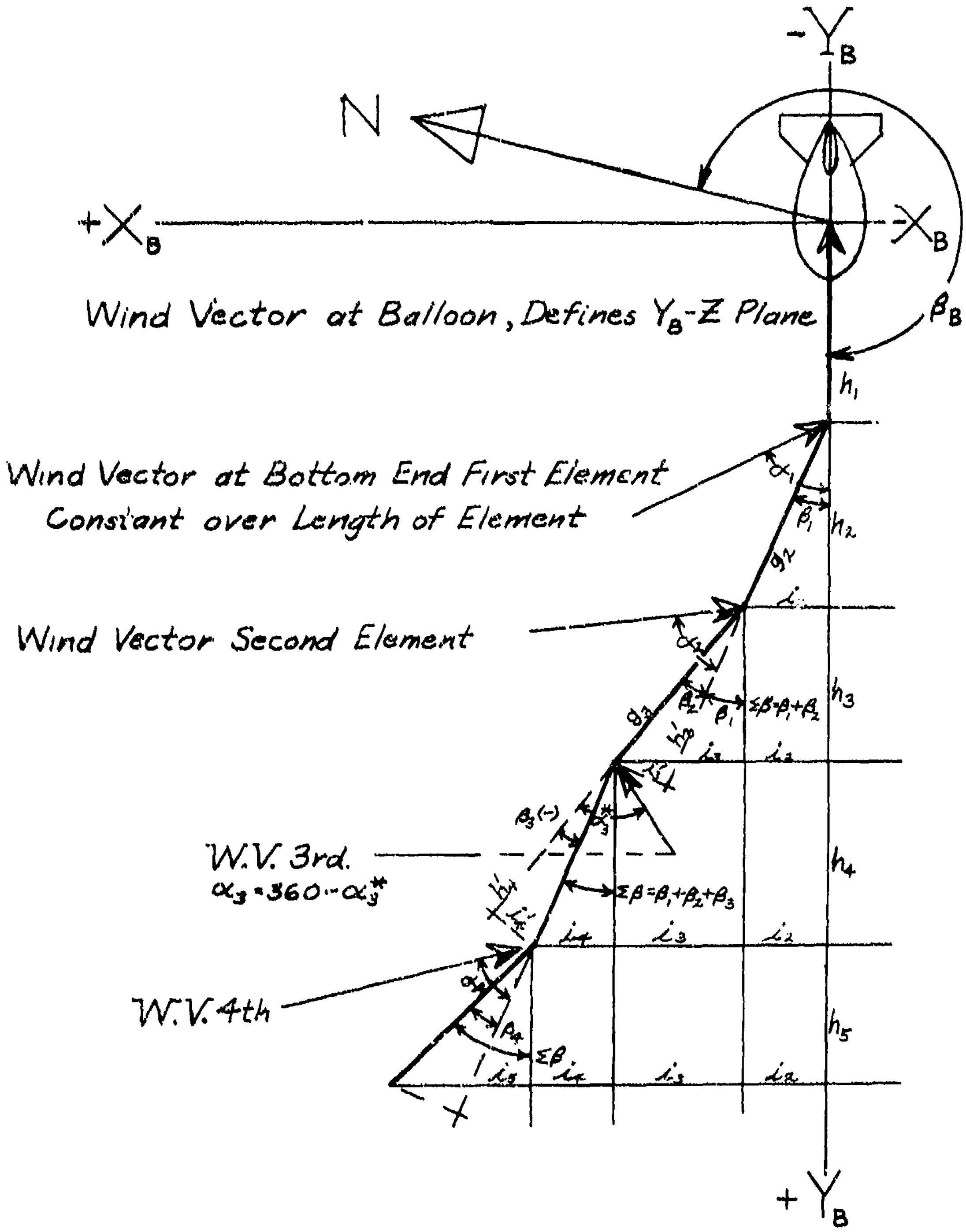


Figure 3. Plan View of the Balloon and the Upper Five Cable Elements

3.2.3 REYNOLDS NUMBER — DRAG COEFFICIENT

Reynolds number is computed by:

$$R = \frac{\rho V_N D}{\mu} . \tag{47}$$

If a constant drag coefficient is not specified, C_D is calculated by one of two methods that are dependent on the value of R. When $R < 1$, the Stokes condition is assumed and

$$C_D = (10.9/R) / (0.87 - \text{Log } R) . \tag{48}$$

When $R > 1$, a series of straight lines are used to approximate the variation of cylinder C_D with R together with

$$C_D = C_{D \text{ Base Point}} + K_R (\text{Log } R - \text{Log } R_{\text{Base Point}}) . \tag{49}$$

The following constants are included in program storage for C_D solution when $R > 1$. The Recall Code Numbers, n_{CD}, are explained in Section 3.2.8.

Recall Code Number	R Region	R_{Base}	$C_{D\ Base}$	K_R
$n_{CD} = 1$	$R > 1 < 9$	1	12.5	− 10.0
$n_{CD} = 2$	$R < 900$	9	2.98	− 1.0
$n_{CD} = 3$	$R < 4500$	900	0.98	0.0
$n_{CD} = 4$	$R < 9000$	4500	0.98	0.7308
$n_{CD} = 5$	$R < 40,000$	9000	1.2	0.0
$n_{CD} = 6$	$R < 50,000$	40,000	1.2	− 4.54
$n_{CD} = 7$	$R < 250,000$	50,000	0.76	0.3434
$n_{CD} = 8$	$R > 250,000$	250,000	1.0	0.0

3.2.4 COEFFICIENT OF VISCOSITY

The Coefficient of Viscosity, μ, is required for the calculation of Reynolds number in Eq. (47). The values of μ in the 1962 Standard Atmosphere can be defined by two straight lines up to 66,000 ft within the accuracy needed here.

From Z = 0 to 36,500 ft: $\mu = 1.205^{-5} - 6.84164^{-11}$ Z lb/ft-sec (50)

From Z = 36,500 to 66,000 ft: $\mu = 0.95528^{-5}$ lb/ft-sec . (51)

3.2.5 DENSITY

The atmospheric density, ρ, is found as a function of altitude, Z, by use of the following equation:

$$\ln \rho/\rho_o = a_o Z + a_1 Z^2 \tag{52}$$

where ρ_o is the density at sea level

$a_o = -2.813606^{-5}$,

$a_1 = -1.77717^{-10}$.

These constants were obtained from a fit of the 1962 Standard Atmosphere up through 70,000 ft with a precision considered satisfactory for this particular application. Additional refinements or use of other atmospheres more typical of seasons or locations of a particular balloon flight could be easily adapted by a change of the two constants.

3.2.6 TERMINATION

The cable element lengths, K, whether entered or automatically made equal to $(Z_B - Z_S) / 100$, are summed in Storage Register No. 024 at each loop in the calculation of end position, tension, etc. When an altitude for the bottom end-point of one cable element is detected below the surface altitude, provision is made to go back to the altitude of the top end-point of that element, divide K by 10—as well as W and A— and then proceed again downward until a bottom end-point goes below Z_S at which time the final printouts occur for the point just above the surface. In effect this process provides a vernier and a solution closer to an exact value at Z_S than possible if K were left unadjusted.

Therefore when resetting for the optional runs at lower balloon altitudes, K, W, and A are multiplied by 10 to reestablish their original values. A vernier of 10 followed by another 10 could be incorporated if an extremely precise surface altitude match were desired. Its need would probably only exist for a zero-wind condition where the cable elevation angle is very large and the error greatest.

Checks are made of both tension and cable elevation angle for positive values before looping back in the program and adding another cable element. Should either not be positive, an appropriate message is delivered and the same final printouts are provided as when a surface condition is reached.

3.2.7 END OF CABLE--WINCH

When the altitude of a bottom end-point of an element reaches the surface, the run is complete and a final printout is provided which represents conditions at the cable winch. Similarly, if the tension should become zero and/or the cable become horizontal, the final printout would represent conditions at the cable end at an altitude above the actual surface altitude. But consider in this discussion that the cable has reached the surface winch.

Figure 4 illustrates a complete idealized balloon-to-winch cable plan view. In working the problem from the balloon downward in altitude, the sign of the Y_B-axis is + ahead of the balloon and the sign of the X_B-axis is + 90° clockwise from the + Y_B-axis. Because of this, a simple transformation of axes to the winch location allows direct application of a conventional sign designation to the winch axes, X_W and Y_W. For example, the sum of the h-distances, H, is positive ahead of the balloon and therefore when referenced to the winch position, this distance would be also positive in the example shown. In all but extreme cases, H would always be positive. However, the distance I can be either + or − depending on relative wind but would also be directly referenced from the winch and its conventional X-Y set of axes.

Therefore, the final output data includes the H and I distances which can be plotted directly as Y and X distances on the conventional set of winch axes to show relative winch-balloon positions. The straight-line distance, L is also printed. By use of the equations shown in Figure 4, the azimuth of the balloon from the winch, AZ_B, is presented. By use of the height, J and the horizontal distance out, L, the elevation angle and slant-range to the balloon are provided.

In addition to the cable tension, length, and weight, the elevation angle or pitch of the cable above the horizontal is also printed. Use of the equation

$$AZ_C = \Sigma\beta + \beta_B - 180° \tag{53}$$

provides the azimuth of the cable leaving the winch.

At this point, machine- or hand-plotted data on tracing paper could be overlaid on a map of the area with the winch point at its known location and the paper rotated to place the north vector on true north if the exact geographic balloon location were desired. To avoid this complexity, a third set of axes was established with the origin at the winch and the winch axes rotated to place the + Y axis on true north. As shown on Figure 4, distances X and Y which can be plotted directly on a map or chart are calculated and are the last two items printed in the final output.

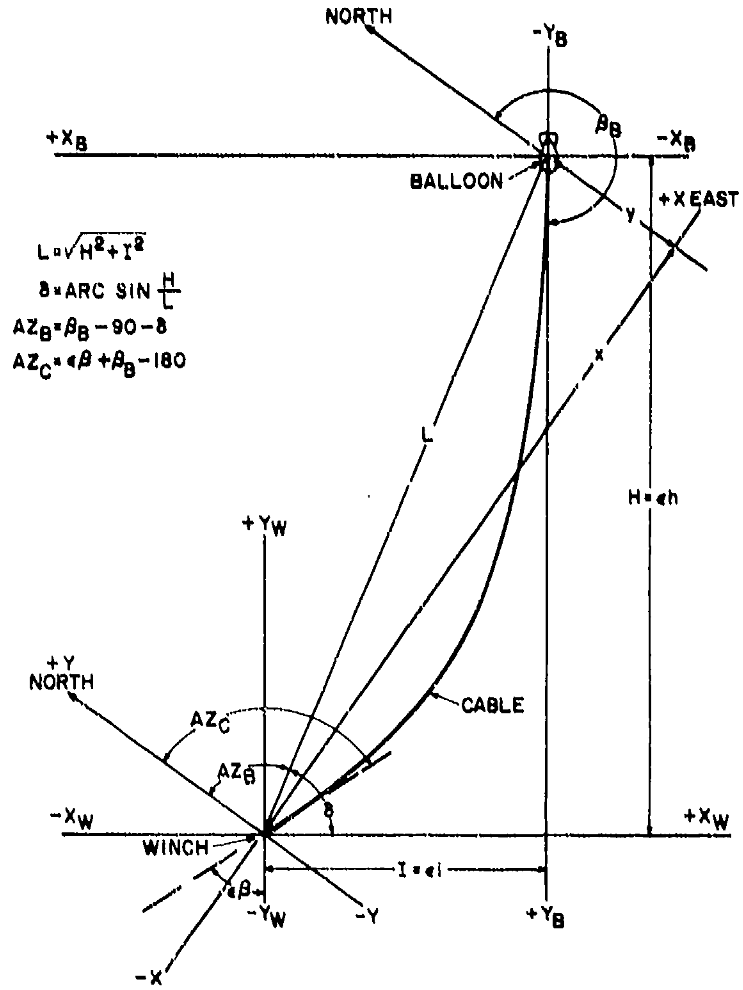


Figure 4. Plan View of a Complete Tether-Cable from Winch to Balloon

3.2.8 HOUSEKEEPING DETAILS

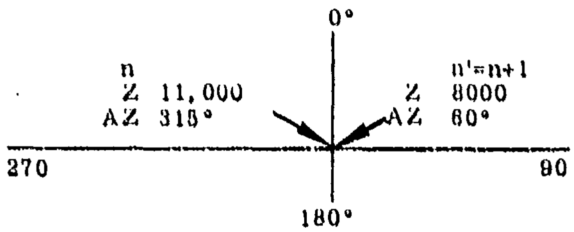
a. Azimuth Management through 0° to 360°

It was decided to keep all specified angles within the 0° to 360° designation whether necessary or not in order that no ambiguity exist particularly on a printout of data. A few examples of detailed handling follows:

When calculating the wind azimuth at a specific altitude, the wind data entered at the nearest altitudes above and below are used to obtain the rate of change of azimuth with altitude (straight-line). As with the wind velocity, the process is downward so that

$$\frac{d\,AZ}{d\,Z_W} = \frac{AZ_{lower} - AZ_{upper}}{Z_{upper} - Z_{lower}} = \frac{AZ_{n+1} - AZ_n}{Z_n - Z_{n+1}}$$

where n = NW = the point number in wind-field table starting with NW = 1 for highest altitude point. Should one azimuth lie in the 4th quadrant and the other be in the 1st quadrant, for example, an improper value of delta azimuth would be determined (azimuth angles are always assumed to lie within the < 180° included angle between the two known azimuths).



$$\frac{d\,AZ}{d\,Z} = \frac{60° - 315°}{11,000 - 8000} = \frac{-255°}{3000}$$

A wind azimuth for Z = 9000 ft would improperly be found in the 2nd quadrant at 145° instead of 25°.

Similarly, reversing the two known points would lead to an improper solution at 9000 ft with 40° instead of 350°.

To handle this problem, the following checks are included in the program logic:

Trial Δ Azimuth, $\Delta AZ_t = AZ_{n+1} - AZ_n$

Is $\Delta AZ_t < 180°$?

No → Is $\Delta AZ_t < -180°$?
- No → $\Delta AZ = \Delta AZ_t$
- Yes → $\Delta AZ = \Delta AZ_t + 360°$

Yes → $\Delta AZ = \Delta AZ_t - 360°$

When the final azimuth is found, an angle outside the 0° to 360° limits might be indicated. To prevent any ambiguity, it is converted by the following process:

Is AZ > 360°?

No | Yes

Is AZ > 0?

No | Yes

AZ = AZ + 360° | AZ = AZ | AZ = AZ − 360° .

A third problem of this nature arises when determining the angle, α, which is the angle between the wind azimuth and the azimuth of the vertical plane containing the cable element. The angle, α, is always defined in the clockwise sense as positive.

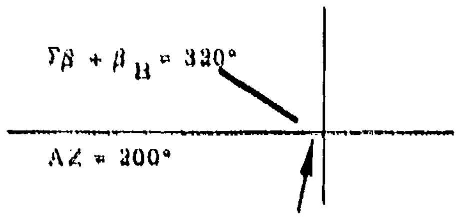
$$\alpha = \text{Wind AZ} - \text{Element AZ}$$

$$\alpha = AZ - (\Gamma\beta + \beta_B)$$

where

$$\beta_B = \text{Wind AZ at } Z_B \text{ or NW} = 1 .$$

If the following were present:



$$\alpha = 200° - 320° = -120° .$$

A positive convention and printout is maintained by:

$$\text{Trial } \alpha,\ \alpha_t = AZ - (\Gamma\beta + \beta_B)$$

Is $\alpha_t < 0°$?

No | Yes

$\alpha = \alpha_t$ | $\alpha = \alpha_t + 360°$.

Thus in the above example, $\alpha = 240°$.

b. Special Problem — Cable Vertical

In the usual tethered-balloon cable system, cable weight is sufficiently large, relative to cable drag, that the cable "sags" or its elevation angle decreases with decreasing altitude. A special case can be found by introducing a large-diameter and very light-weight cable into a strong wind field which causes the cable to

increase its elevation angle through 90° at some altitudes. When this happens, an ambiguity in the equation utilized causes a sudden and false reversal to a negative tension and, if uncorrected, a readout that the tension is zero. To take care of this special case, a subroutine (LABEL π) is introduced which may be examined in the flow charts. It essentially corrects the computed angle β by adding 180° to β when the cable goes through the vertical. The physical meaning of this correction is apparent in the horizontal plane (H-I) where θ goes through 90°, the projection of the cable has a reflex of 180° change of direction. (See Figure 20).

c. Storage and Recall Codes

(1) Drag Coefficient

The eight points defining the drag coefficient of a cylinder, when Reynolds Number is greater than 1.0, each consists of three constants as described in Section 3.2.3. They are stored in Storage Register No. 077 through 100.

Recall of the parameters is made by use of Code Number called n_{CD}. A value of 1 is assigned for the group of parameters defined by the smallest R or Registers No. 077, 078, and 079 followed by a value of two for the next group and on up to eight. Each of the parameters are extracted when the proper R area is found by the use of indirect addressing utilizing these formulae:

$$3\,n_{CD} + 74 \text{ for } R_B \,,$$

$$3\,n_{CD} + 75 \text{ for } C_{DB} \,,$$

$$3\,n_{CD} + 76 \text{ for } K_R \,.$$

(2) Wind

The wind profile may be defined by as few as two altitude points or as many as 12. Each point consists of three user-entered values: altitude, wind velocity, and azimuth. For both entry and recall, use is made of a Code Number called NW.

For storage, use is made of indirect addressing and the following:

$$3\,NW + 36 \text{ for Altitude} \,,$$

$$3\,NW + 37 \text{ for Wind Velocity} \,,$$

$$3\,NW + 38 \text{ for Azimuth} \,.$$

The value of NW begins at 1 for the first group of entries or highest altitude and increases by 1 at each lower point down to and including the surface. Following entry of all points in the wind field a value of NW = 1 is reassigned since the computation process starts at the highest altitude and works downward.

Recall of the parameters in Program No. 76.006 was made by the same formula listed above. In these programs however, the optional rerun cycles with the balloon at successively lower altitudes listed in the wind table required a modification. A Repeater Code Number, r, is used herein for several purposes. A value of 0 is used during the primary runthrough with the balloon at maximum altitude (Run No. 1). If the option rerun cycles are called for, r is indexed + 3 for each successive run at lower altitudes. The printed Run Number is found by:

$$\text{Run Number} = 1 + \frac{r}{3}.$$

Recall of the wind parameters is made by indirect addressing of the following formula:

3 NW + 36 + r for Altitude,

3 NW + 37 + r for Wind Velocity,

3 NW + 38 + r for Azimuth .

3.2.9 WIND PROFILE

The usual available wind data consists of the wind magnitude in knots and its azimuth (direction from which the wind is blowing) in degrees from true North at various altitudes above the surface. For a low altitude balloon, the twelve available altitude—wind storage groups in the programs are more than enough to handle the typical amount of wind data available.

For higher altitudes where more than twelve levels of wind information is available,some editing, smoothing, or averaging may be necessary. The magnitude and azimuth may each be plotted and points defining significant changes in each parameter be used to define significant altitude levels. As an example, in Figure 5, the wind magnitude and azimuth are plotted as points. Some points can be usually ignored as dubious or offering little effect on the total wind picture. Significant points defining the wind magnitude variation with altitude as a series of six straight lines are found at the following altitudes:

28,000	13,500
23,000	6500
17,000	3000
	0 .

Significant points defining the wind azimuth variation with altitude as a series of seven straight lines are found at the following altitudes:

28,000	9000
25,000	3000
22,500	0 .
17,000	
15,000	

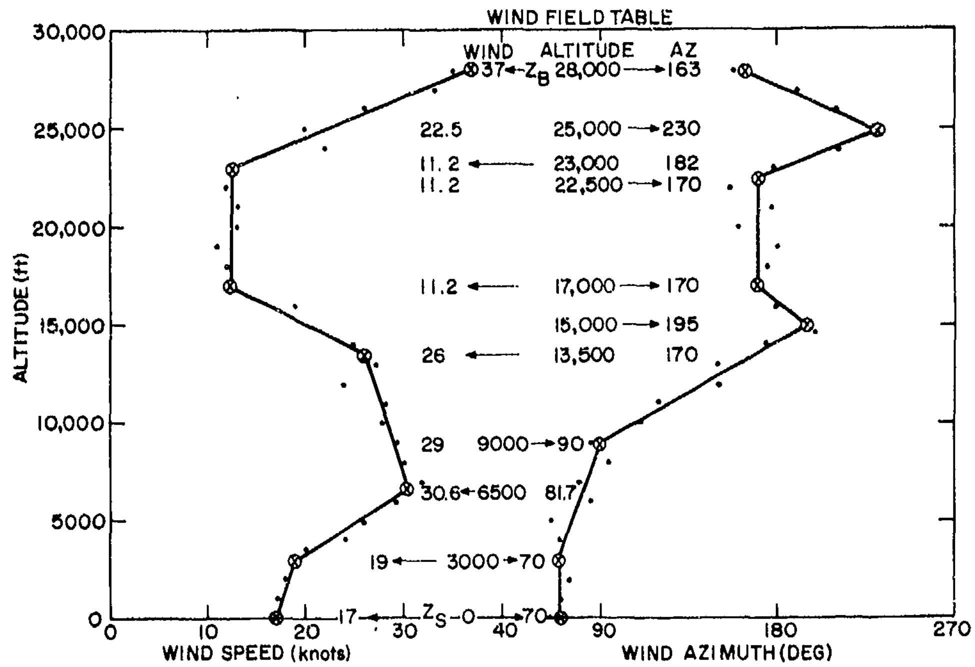


Figure 5. Typical Wind Information

Since four of these are the same altitudes defining the wind magnitude, a total of eleven altitudes are sufficient to define the magnitude and azimuth variation as shown by the Table on Figure 5.

The use of straight line interpolations between entered wind magnitudes and between entered azimuths for each intermediate altitude is based on the simplicity of this type of operation and, in effect, the restructuring of the original averaged wind profile described above. It does not pretend to reflect the best meteorological technique for precise wind analysis. However, if reasonable care and judgement is used in selecting the input points defining the two parameters, the resulting output cable position and conditions will be within the acceptable engineering standards used throughout the program.

As described in Sections 4. 1, A and B, two adjacent input wind points should never include azimuth angles that are exactly 180° apart. In order to avoid an ambiguity as to which way a wind field is rotating between altitudes in such rare cases, altitudes should be selected so that adjacent azimuth angles differ by less than 180°.

3.2.10 MAGNETIC PROGRAM AND DATA CARDS

A. Program Cards

Program No. 77.007 and No. 77.007P each require two HP No. 9162-0012 magnetic program cards utilizing four sides each program. As explained in their operating instructions, several versions of each program might be desirable. For example the following three sets of cards for each program have been found useful in avoiding excessively long rolls of output data when it is not required:

(1)	Print no intermediate altitude data from balloon to surface	plus	Print final surface data for cable condition at the winch,
(2)	Print only Z, H, I, θ, and T values from balloon to surface	plus	Print final surface data for cable condition at the winch,
(3)	Print all intermediate data from balloon to surface	plus	Print final surface data for cable condition at the winch.

Program No. 77.007B also requires four-sides of two program cards. However, no printing of intermediate altitude data is provided (see Section 3.4.1).

B. Data Cards

In order to conserve program steps, in the development of the programs, twenty-six drag coefficient and density constants were stored on magnetic data cards. Therefore, two HP No. 9162-0012 cards (three sides) are required to place the constants in the registers chosen. As explained in the operating instructions, these cards are loaded into the machine in the process of loading the program cards and the constants will be retained through any number of problem solutions until the machine is turned off. One set of data cards can be used interchangeably with Program No. 77.007 or Program No. 77.007P; Program No. 77.007B receives the constants through Program No. 77.007.

The two-page listing of the data program, which follows hereunder, inserts the constants into the proper storage registers and then provides a printout of all storage register numbers and contents for review. Register numbers 000 through 074 and 101 through 108 should contain zeros. After reviewing the contents of register numbers 075 through 100 for accuracy, the data cards can be recorded by the following key strokes:

END

FMT

x →

Alternately, if data cards are not used,the constants may be loaded into storage directly by key punch each time the program is loaded. The following step numbers should be changed as shown to prevent automatic Insert Card light and loading motor turn-on.

	No. 77.007 Step Nos.	No. 77.007P Step Nos.
Change contents to CNT:	0040, 1, and 2	0041, 2, and 3

Entry of Constants into Storage Registers With Review of Entire Register Contents for Program Nos. 77.007 and 77.007P Data Cards (Sheet 1 of 2)

STEP	KEY	CODE	x	y	z	STEP	KEY	CODE	x	y	z
0000	CLR					0	5		12.5		
1	TAB					1	X→				
2	CLRX					2	0				
3	1					3	7				
4	•					4	8				
5	7					5	1				
6	7					6	0				
7	7					7	Chg S		−10		
8	1					8	X→				
9	7					9	0				
0	Chg S					0	7				
1	E EXP					1	9				
2	1					2	9		9		
3	0					3	X→				
4	Chg S		a_1			4	0				
5	X→					5	8				
6	0					6	0				
7	7					7	2				
8	5					8	•				
9	2					9	9				
0	•					0	8		2.98		
1	8					1	X→				
2	1					2	0				
3	3					3	8				
4	6					4	1				
5	0					5	9				
6	6					6	0				
7	Chg S					7	0		900		
8	E EXP					8	X→				
9	5					9	0				
0	Chg S		a_0			0	8				
1	X→					1	3				
2	0					2	•				
3	7					3	9				
4	6					4	8		.98		
5	1		1			5	X→				
6	X→					6	8				
7	7					7	4				
8	7					8	X→				
9	X→					9	0				
0	9					0	8				
1	9					1	7				
2	Chg S		−1			2	4				
3	X→					3	5				
4	0					4	0				
5	8					5	0		4500		
6	2					6	X→				
7	1					7	0				
8	2					8	8				
9	•					9	6				

Entry of Constants into Storage Registers With Review of Entire Register Contents for Program Nos. 77.007 and 77.007P Data Cards (Sheet 2 of 2)

STEP	KEY	CODE	X	Y	Z	STEP	KEY	CODE	X	Y	Z
0100 0	•					0	7				
1	7					1	6		.76		
2	3					2	X→				
3	0					3	0				
4	8		.7308			4	9				
5	X→					5	6				
6	0					6	•				
7	8					7	3				
8	8					8	4				
9	9					9	3				
0	EExp					0	4		.3434		
1	3		9000			1	X→				
2	X→					2	0				
3	0					3	9				
4	8					4	7				
5	9					5	2				
6	1					6	•				
7	•					7	5				
8	2		1.2			8	EExp				
9	X→					9	5		250000		
0	9					0	X→				
1	0					1	0				
2	X→					2	9				
3	0					3	8				
4	9					0174	a		Reg. No		
5	3					5	PNT				
6	4					6	X( )				
7	EExp					7	IND				
8	4		40000			8	a		Content		
9	X→					9	PNT				
0	0					0	PNT				
1	9					1	1		1		
2	2					2	X→				
3	4					3	+				
4	•					4	a				
5	5					5	a		Reg. No		
6	4					6	↑			Reg. No.	
7	ChgS		-4.54			7	1				
8	X→					8	0				
9	0					9	9		109	Reg No	
0	9					0	X>y				
1	4					1	0				
2	5					2	1				
3	EExp					3	7				
4	4		50000			4	4				
5	X→					5	END				
6	0					6					
7	9					7					
8	5					8					
9	•					9					

3.3 **Program No. 77.007**

This, the basic version of the three-dimensional tether-cable program provides only printed outputs. Therefore the MATH ROM and a PRINTER-ALPHA ROM only need be installed in the HP 9810A Calculator. If plots are desired, Program No. 77.007P should be used.

3.3.1 ACCESS TO PROGRAM NO. 77.007B

As explained in Section 3.3.2 and under Section 3.4, this program, 77.007, must be used as a first step in entering Program No. 77.007B. Once tied-in, these two programs can be called back and forth for solution of many detailed problems.

3.3.2 RERUN OPTIONS

When a problem is solved and the cable end conditions printed, the program is designed to offer the user a choice of methods with which to proceed to the next problem. A message is printed:

OPT.ENT (OPTION, ENTER)

followed by a choice of four numbers and a STOP.

When a completely new problem requiring reentry of different altitudes, winds, and cable parameters is next to be run, the number 0 is entered. The program will clear all but the storage registers loaded from the data cards and cycle back to the initial printing of the program number and title.

There are situations where a series of problems involve only changes in the cable specifications, the element length, or the balloon total force and its angle. For this situation, where the altitudes and the wind field table remain unchanged from one problem to another, the entry of the number 1 in the above STOP will save these parameters unchanged from the previous problem. The program will clear only summation registers and cycle back to the initial printing of the program number and title. The STOPS normally used for entry of altitudes and the wind field will be by-passed although these saved parameters will be printed at their proper locations.

The normal runs (printed RUN#1 MAX ALT) provide a cable solution; (a) for the balloon at a specific altitude subjected to the winds specified in the first group of wind entries, and (b) for the cable subjected to a variety of wind conditions specified at the lower altitudes down to the surface. One practical and sometimes limiting problem is concerned with raising or lowering the balloon through the same specified wind field. Rather than a reentry of all variables required at each of many lower altitudes, a method is provided to simplify a series of runs at decreasing altitudes.

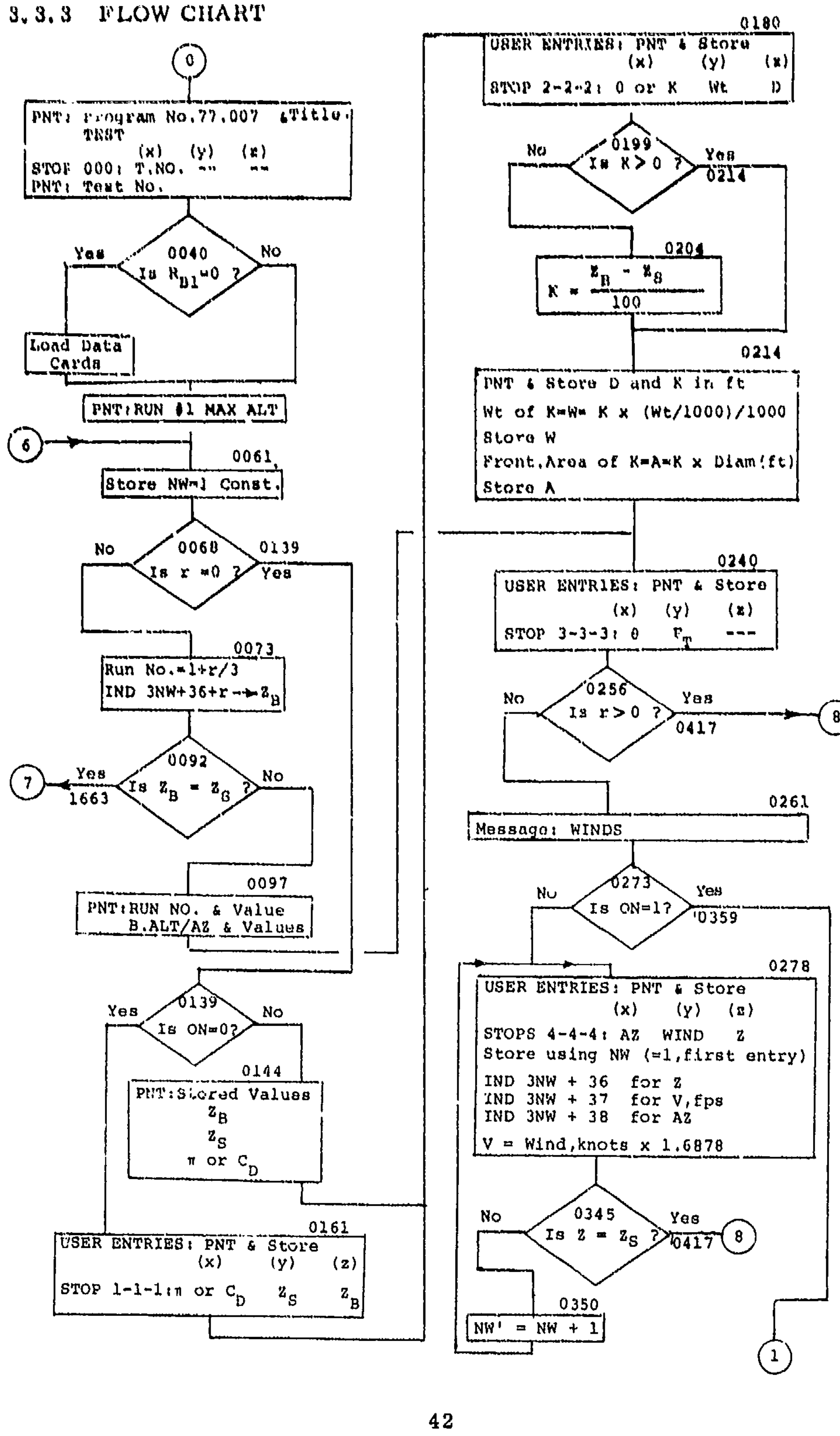
If the number 2 is entered at the above STOP, the program will clear the summation registers and retain the surface altitude, cable specifications, and wind field, and setup RUN#2 with the balloon at the second lower altitude in the wind

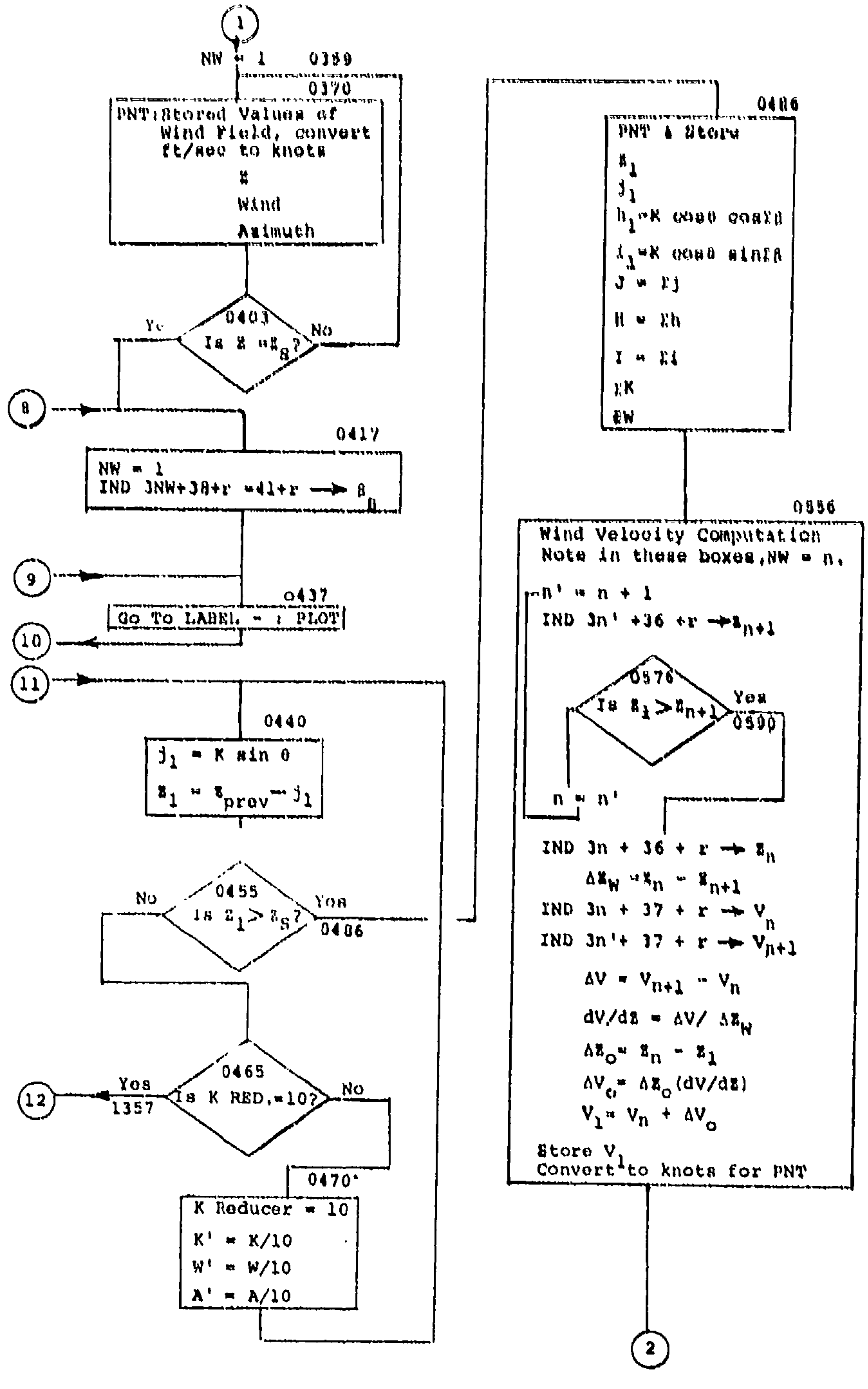
field table. Because the balloon total force and angle at this new altitude will differ from the previous balloon altitude, only one STOP is needed for these two entries. A full solution is then provided for this condition. At its end, the program loops to RUN#3 and places the balloon at the third lower altitude in the wind field table. These automatic loops continue until the surface is reached. At that point the OPT. END message is printed but a choice of only 0 or 1 is then permitted.

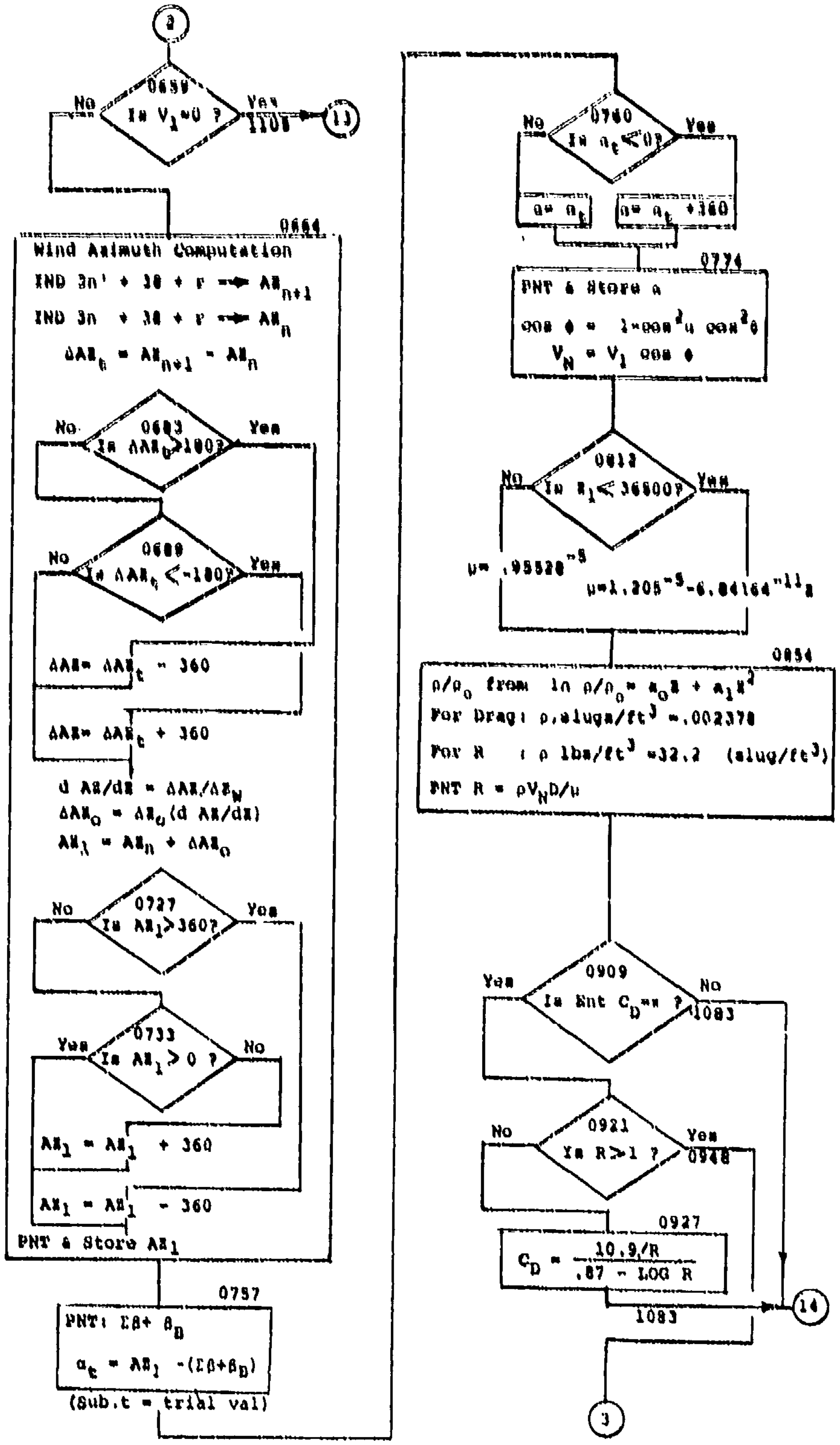
When considering the use of this lowering altitude cycle option, it is necessary to initially enter a wind field table which specifies a sufficient number of altitudes to provide a complete performance analysis. Altitudes having particularly strong winds, of concern in balloon performance, would of course be included in the initial definition of the wind field acting on the cable. Similarly, it is necessary to have the values of the balloon total force, F_T, and its angle, θ, available for entry at each altitude as indicated in the INPUT DATA FORM. The use of Program No. 76.003 or No. 76.005, Reference 1, for each of the altitudes in the wind field table will provide the values of F_T and θ.

The effect of a change in wind on the balloon cable system with the just-calculated cable length held fixed is a practical problem. This question can be answered by use of the number 8 entered at the above OPT ENT STOP -- only after an original, OPT 0, or OPT 1 solution. As the message indicates, this requires the use of Program No. 77.007B whose cards should be ready for loading when selecting this option. If this replacement by 77.007B is made, readmission to 77.007 can be easily made according to the directions contained under 77.007B (see Section 5.4). Data cards are not needed in these interchanges as the constants involved are protected by program logic.

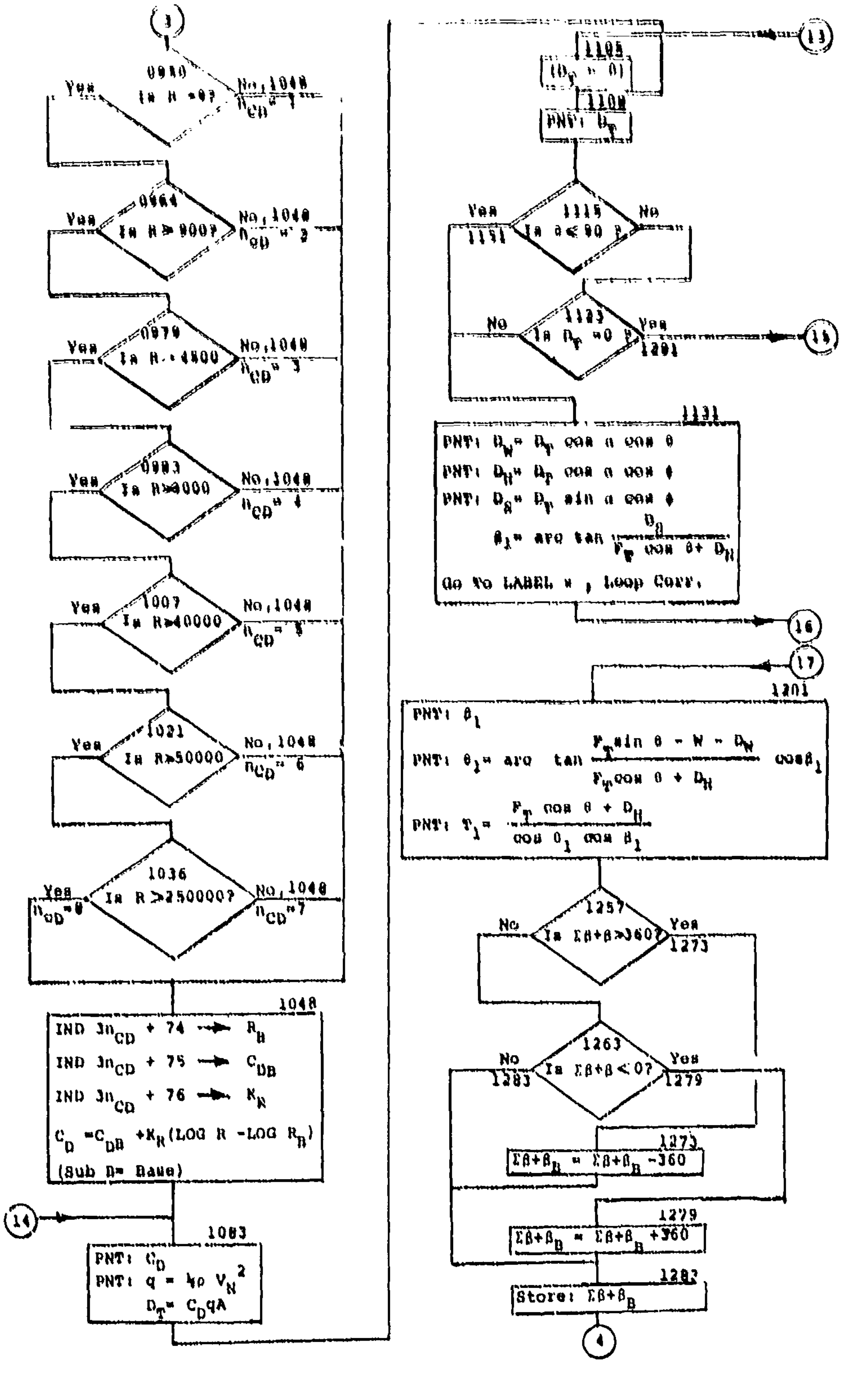
3.3.3 FLOW CHART

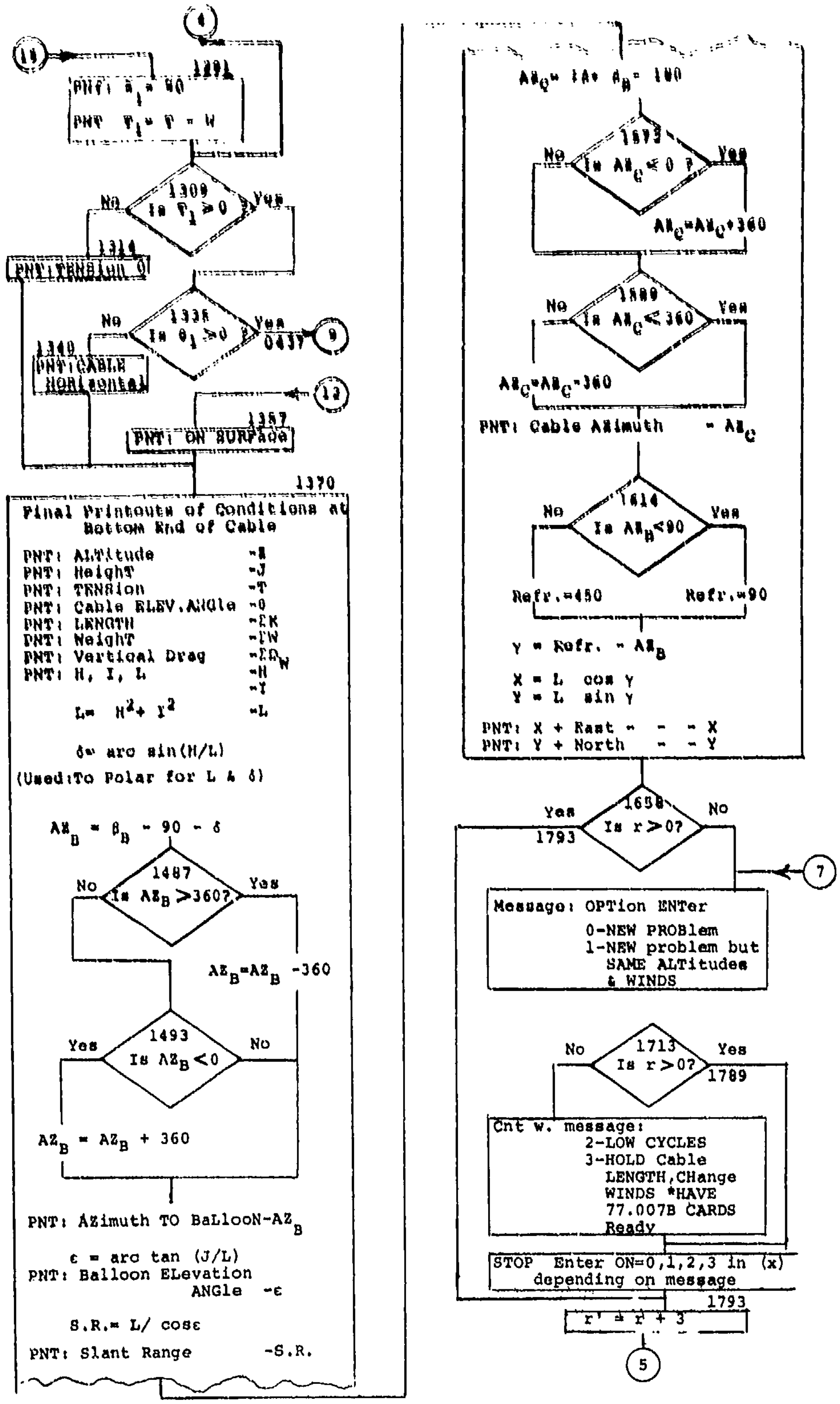



1
NW = 1 0369
0370
PNT: Stored Values of Wind Field, convert ft/sec to knots
Z
Wind
Azimuth
0403
Is Z = Z8?
Yes
No
8
0417
NW = 1
IND 3NW+38+r =41+r → 8
9
0437
Go To LABEL - : PLOT
10
11
0440
j1 = K sin θ
Z1 = Zprev - j1
0455
Is Z1 > ZS?
No
Yes
0486
0465
Is K RED. = 10?
Yes
1357
No
12
0470
K Reducer = 10
K' = K/10
W' = W/10
A' = A/10
0486
PNT & Store
Z1
j1
h1 = K cosθ cosΣβ
i1 = K cosθ sinΣβ
J = Σj
H = Σh
I = Σi
ΣK
ΣW
0556
Wind Velocity Computation
Note in these boxes, NW = n.
n' = n + 1
IND 3n' +36 +r → Zn+1
0576
Is Z1 > Zn+1
Yes
0590
n = n'
IND 3n + 36 + r → Zn
ΔZW = Zn - Zn+1
IND 3n + 37 + r → Vn
IND 3n' + 37 + r → Vn+1
ΔV = Vn+1 - Vn
dV/dZ = ΔV/ ΔZW
ΔZo = Zn - Z1
ΔVo = ΔZo (dV/dZ)
V1 = Vn + ΔVo
Store V1
Convert to knots for PNT
2

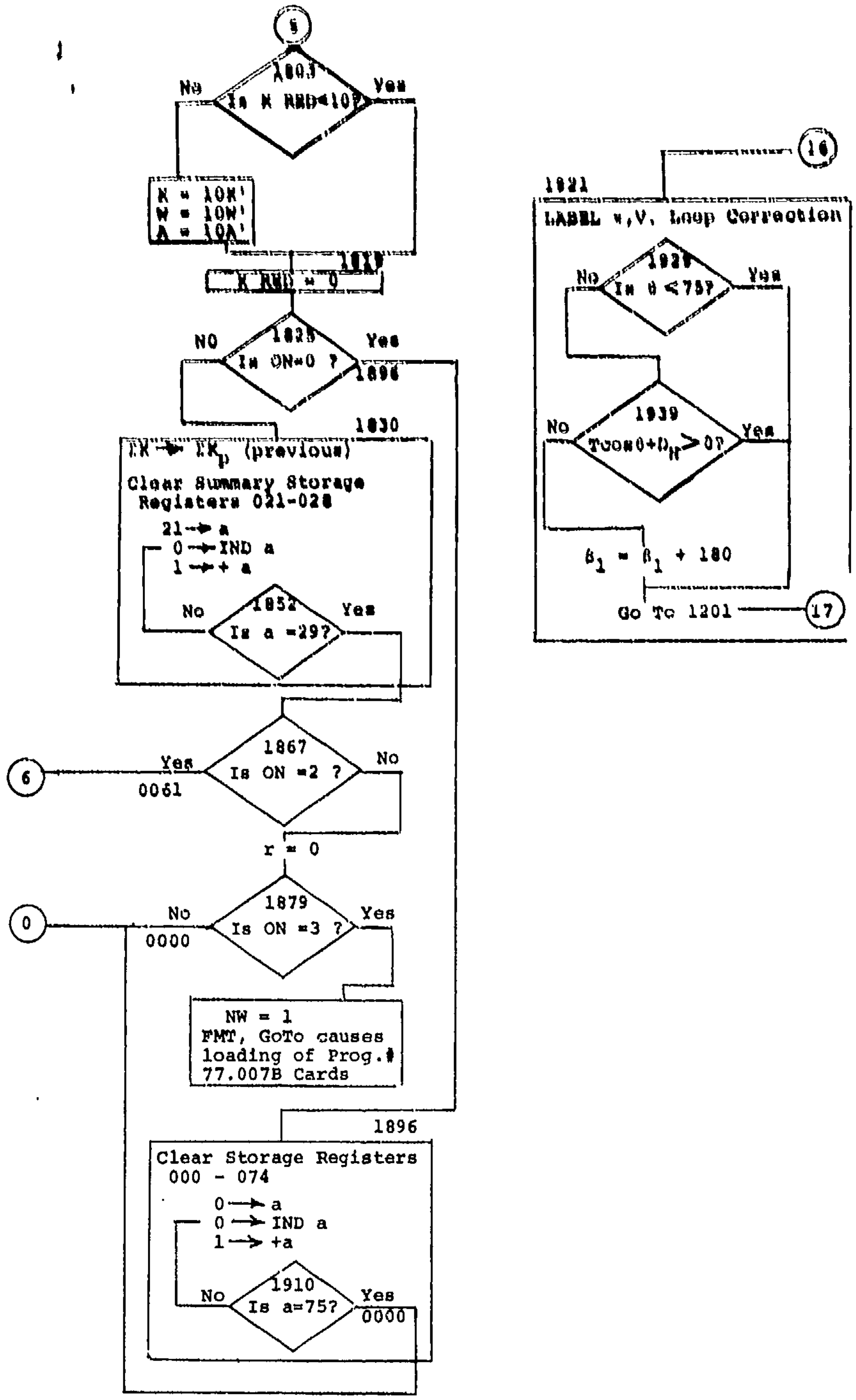


2
0659
Is V1=0 ?
No
Yes
1105
11
0664
Wind Azimuth Computation
IND 3n' + 38 + r → AZn+1
IND 3n + 38 + r → AZn
ΔAZt = AZn+1 - AZn
0683
Is ΔAZt>180?
No
Yes
0689
Is ΔAZt <-180?
No
Yes
ΔAZ= ΔAZt - 360
ΔAZ= ΔAZt + 360
d AZ/dZ = ΔAZ/ΔZN
ΔAZO = AZO(d AZ/dZ)
AZ1 = AZn + ΔAZO
0727
Is AZ1>360?
No
Yes
0733
Is AZ1 > 0 ?
Yes
No
AZ1 = AZ1 + 360
AZ1 = AZ1 - 360
PNT & Store AZ1
0757
PNT: Σβ+ βB
αt = AZ1 -(Σβ+βB)
(Sub.t = trial val)
0760
Is αt<0?
No
Yes
α= αt
α= αt +360
0774
PNT & Store α
cos φ = 1-cos²α cos²θ
VN = V1 cos φ
0812
Is Z1< 36600?
No
Yes
μ= .95528^-5
μ=1.205^-5-6.84164^-11Z
0854
ρ/ρ0 from ln ρ/ρ0= a0Z + a1Z²
For Drag: ρ,slugs/ft³ =.002378
For R : ρ lbs/ft³ =32.2 (slug/ft³)
PNT R = ρVND/μ
0909
Is Ent CD=* ?
Yes
No
1083
0921
Is R>1 ?
No
Yes
0948
0927
CD = 10.9/R / .87 - LOG R
1083
14
3


0980
Is R =0?
Yes
No, 1048
n_{CD} = 1
0964
Is R > 900?
No, 1048
n_{CD} = 2
0979
Is R > 4800
No, 1048
n_{CD} = 3
0993
Is R>9000
No, 1048
n_{CD} = 4
1007
Is R>40000
No, 1048
n_{CD} = 5
1021
Is R>50000
No, 1048
n_{CD} = 6
1036
Is R >250000?
n_{CD} = 8
No, 1048
n_{CD} = 7
1048
IND $3n_{CD}$ + 74 → R_B
IND $3n_{CD}$ + 75 → C_{DB}
IND $3n_{CD}$ + 76 → K_R
$C_D = C_{DB} + K_R(\text{LOG } R - \text{LOG } R_B)$
(Sub B= Base)
14
1083
PNT: C_D
PNT: $q = \frac{1}{2}\rho V_N^2$
$D_T = C_D qA$
13
1105
$(D_T = 0)$
1108
PNT: D_T
1115
Is $\theta \leq 90$?
Yes
No
1131
1123
Is $D_T = 0$?
No
Yes
1201
15
1131
PNT: $D_W = D_T \cos\alpha \cos\theta$
PNT: $D_H = D_T \cos\alpha \cos\phi$
PNT: $D_S = D_T \sin\alpha \cos\phi$
$\beta_1 = \text{arc tan} \frac{D_S}{F_T \cos\theta + D_H}$
Go To LABEL x, Loop Corr.
16
17
1201
PNT: β_1
PNT: $\theta_1 = \text{arc tan} \frac{F_T \sin\theta - W - D_W}{F_T \cos\theta + D_H} \cos\beta_1$
PNT: $T_1 = \frac{F_T \cos\theta + D_H}{\cos\theta_1 \cos\beta_1}$
1257
Is $\Sigma\beta+\beta > 360$?
No
Yes
1273
1263
Is $\Sigma\beta+\beta < 0$?
No
1283
Yes
1279
1273
$\Sigma\beta+\beta_B = \Sigma\beta+\beta_B - 360$
1279
$\Sigma\beta+\beta_B = \Sigma\beta+\beta_B + 360$
1283
Store: $\Sigma\beta+\beta_B$
4



1301
PNT: R_1 = RO
PNT T_1 = T - W
1309
Is T_1 > 0 ?
No
Yes
1314
PNT: TENSion 0
1335
Is θ_1 > 0 ?
No
Yes
0437
1340
PNT: CABLE HORizontal
1357
PNT: ON SURFace
1370
Final Printouts of Conditions at Bottom End of Cable
PNT: ALTitude -Z
PNT: HeighT -J
PNT: TENSion -T
PNT: Cable ELEV.ANGle -θ
PNT: LENGTH -ΣK
PNT: WeighT -ΣW
PNT: Vertical Drag -ΣD_W
PNT: H, I, L -H -I
L = H² + I² -L
δ = arc sin(H/L)
(Used: To Polar for L & δ)
AZ_B = β_B - 90 - δ
1487
Is AZ_B > 360?
No
Yes
AZ_B = AZ_B - 360
1493
Is AZ_B < 0
Yes
No
AZ_B = AZ_B + 360
PNT: Azimuth TO BaLlooN-AZ_B
ε = arc tan (J/L)
PNT: Balloon ELevation ANGle -ε
S.R. = L/ cosε
PNT: Slant Range -S.R.
AZ_C = IA + β_B - 180
1572
Is AZ_C < 0 ?
No
Yes
AZ_C = AZ_C + 360
1589
Is AZ_C < 360
No
Yes
AZ_C = AZ_C - 360
PNT: Cable AZimuth - AZ_C
1614
Is AZ_B < 90
No
Yes
Refr. = 450
Refr. = 90
γ = Refr. - AZ_B
X = L cos γ
Y = L sin γ
PNT: X + East - - - X
PNT: Y + North - - Y
1658
Is r > 0?
Yes
No
1793
Message: OPTion ENTer
0-NEW PROBlem
1-NEW problem but SAME ALTitudes & WINDS
1713
Is r > 0?
No
Yes
1789
Cnt w. message:
2-LOW CYCLES
3-HOLD Cable LENGTH, CHange WINDS *HAVE 77.007B CARDS Ready
STOP Enter ON=0,1,2,3 in (x) depending on message
1793
r' = r + 3



5
No
1803
Yes
Is K RND<10?
K = 10K'
W = 10W'
A = 10A'
K RND = 0
NO
1825
Yes
Is ON=0 ?
1896
1830
(previous)
Clear Summary Storage
Registers 021-028
21→ a
0 → IND a
1 → + a
No
1852
Yes
Is a =29?
1867
Yes
No
Is ON =2 ?
6
0061
r = 0
0
No
1879
Yes
Is ON =3 ?
0000
NW = 1
FMT, GoTo causes
loading of Prog.#
77.007B Cards
1896
Clear Storage Registers
000 - 074
0 → a
0 → IND a
1 → +a
No
1910
Yes
Is a=75?
0000
16
1921
LABEL Loop Correction
No
Yes
1939
No
Yes
Go To 1201
17

3.3.4 OPERATING INSTRUCTIONS AND NOTES

KEY STROKES	ENTRIES
RUN	
END	
FIX 2 , 3	---(Number of decimal places desired)
	Insert Side 1 of Program Cards
LOAD	
	Continue inserting Sides 2, 3, and 4 of Program Cards
	Insert Side 1 of Data Cards
END	
CONT	
	(Following heading is now printed) PROG # 77.007 3-DIM. TETHER TEST

	(X)	(Y)	(Z)
At STOP 0-0-0, Enter:	Test No/I.D.	———	———

CONT

Following printing of the test number or I.D., Side 1 of the Data Card will load.

Continue inserting Sides 2 and 3 of Data Cards

Note that loading of Program and Data Cards will only be required one time until machine is turned off.

(Following is now printed)

RUN #1 MAX ALT

At STOP 1-1-1, Enter:	π or C_D *	Z_S (ft, MSL)	Z_B (ft, MSL)

*Enter π to use built-in cylinder C_D variation or enter C_D value which will be held constant throughout program run.

CONT

At STOP 2-2-2, Enter:	0 or K* (Element Length)	Wt/1000 ft (lbs) (Cable)	Diam (in.) (Cable)

*Enter 0 to set $K = \frac{Z_B - Z_S}{100}$ or enter K in ft

CONT

At STOP 3-3-3, Enter:	θ (deg) (Angle of F_T to Horizon)	F_T (lb) (Balloon Tot. Force)	———

CONT

WINDS (Printed)

At STOP 4-4-4, Enter:	Azimuth of wind (deg)	Wind (knots)	Z_B (ft MSL)

This entry must be for conditions at balloon altitude.

CONT

At STOP 4-4-4, Enter:	Azimuth	Wind	Z

Stop 4's will repeat until the last set of entries for the surface condition at Z_S are inserted. A total of 12 sets of wind entries may be made including the balloon and surface conditions

CONT

The above sets of entries will be printed out in groups when CONT is struck after Stops 1, 2, 3, and 4. The entered or computed value will be printed in the case of K.

The program now begins computation starting at the balloon and works downward one element at a time to the surface. The following twenty-three parameters are printed for the condition at the bottom end-point of each element. NOTE: To avoid the printout of any or all of the twenty-three parameters shown below, replace PNT with CNT at the associated program step numbers.

Step No.		
0488	Z	Altitude, ft MSL
0490	j	Vert. Distance, top to bottom of element, ft
0513	h	Horiz. Distance, parallel to balloon axis, ft
0524	i	Horiz. Distance, perpendicular to balloon axis, ft
0532	J	Total Vert. Distance, balloon to bottom of element, ft
0536	H	Total Horiz. Dist., Σh, balloon to bottom of element, ft
0540/1	I	Total Horiz. Dist., Σi, balloon to bottom of element, ft
0545	ΣK	Total Element (Cable) Length, ft
0555	ΣW	Total Element (Cable) Weight, lb
0656	Wd	Wind Velocity at bottom of element, knots
0755	AZ	Wind Azimuth at bottom of element, deg
0757	$\Sigma\beta + \beta_B$	Azimuth of Vert. Plane Containing the Element, deg
0778	α	Wind Incidence Angle on the Element, deg
0902	R	Reynolds Number
1084	C_D	Drag Coefficient
1098	q	Dynamic Pressure, lb/ft^2
1108	D_T	Total Element Drag, lb
1147	D_W	Vertical Drag Component, lb
1159	D_H	Horiz. Drag Comp. in Vertical Plane of the element, lb
1173/4	D_S	Horiz. Drag Comp. to Vertical Plane of the element, lb
1201	β_1	Horiz. Rotation of Tension Vector at bottom end or Horiz. Rotation of the next element's vertical plane, deg.
1235 1293	θ_1	Pitch Angle Downward of Tension Vector at bottom end or Elevation angle of next element above the horizon, lb
1248/9 1303/4	T_1	Tension at bottom end or at top end of next element, lb

Groups of the values of the above 23 parameters will continue to be printed for points down the cable until one of the following conditions is encountered:

(a) The cable reaches the earth's surface at Z_S—the winch location.

(b) The tension becomes zero.

(c) The cable becomes horizontal.

(In (a), the computational techniques used do not yield a precise Z_S condition. The final Z will be higher than Z_S by an amount less than K sin $\theta/10$, usually no more than a few feet).

The final printout includes the abbreviated names and values of the following parameters. They describe the conditions at the winch if condition (a) is attained or at the cable lower-end which is above the surface if conditions (b) or (c) are indicated.

(a) ON SURFACE or (b) TENSION, 0 or (c) CABLE HOR

ALT	Z	Altitude, ft
HT	J	Vertical Height, ft
TENSION	T	Cable Tension, lb
C. ELEV. ANG	θ	Elevation Angle of Cable above Horizon, deg
LENGTH	ΣK	Cable Length, ft
WT	ΣW	Cable Weight, lb
V. DRAG	ΣD_W	Total Vertical Drag Component, lb
H, I, L	H	Tot. Horiz. Distance along Y_B or Y_W axis, ft
	I	Tot. Horiz. Distance along X_B or X_W axis, ft
	L	Min. Direct Horizontal Dist. to Balloon, ft
AZ. TO BLN	AZ_B	Azimuth Angle to Balloon, deg
B EL. ANG	ϵ	Elevation Angle to Balloon, deg
S. R	SR	Slant Range to Balloon, ft
CABLE AZ	AZ_C	Azimuth Angle of Cable (Out of Winch), deg
X +E	X	X Coordinate to Balloon, ft
Y +N	Y	Y Coordinate to Balloon, ft

At this point the initial problem entered is solved with printing completed. If this was an initial run (RUN #1 MAX. ALT), the following is printed and STOP provided to permit 3 optional ways to rerun the program.

```
OPT. ENT
0 - NEW PROB
1 - NEW-SAME ALTS /
    WINDS
2 - LOW CYCLES
3 - HOLD C. LENGTH
    CHG. WINDS* HAVE
    77.007B CARDS
    READY
```

At STOP, Enter: 0, 1, 2, or 3 in (X)

$\boxed{\text{CONT}}$

If 0 is Entered: New Problem—Use for completely new problem. All but the permanent storage registers containing density and drag coefficient constants will be cleared, program returns to start with reprint of Number and Title.

If 1 is Entered: New Problem—but Same Altitudes and Winds—Use when only the cable diameter, cable weight, element length, or balloon force and angle is to be changed in next problem. Summary storage registers will be cleared and program returns to start with reprint of Number and Title. Program proceeds on with only STOPS 2 and 3 for associated entries. STOPS 1 and 4 are not activated but the altitude and wind field data will be automatically printed at the proper locations from storage registers loaded from the previous problem.

If 2 is Entered: Lower Altitude Cycles—Use when analysis is desired with the balloon lowered to each of the altitudes specified in the wind field table. Summary storage registers are cleared and the program returns to start with reprint of Number and Title. Instead of RUN #1 MAX. ALT the following is printed:

RUN #

2

B. ALT/AZ

Z_B

β_B

Program then goes to STOP 3 for entry of balloon total force and angle at this new altitude Z_B which is the second altitude originally entered in the wind table. The program will then make a complete solution to the surface (or to zero tension or horizontal cable) for this new balloon altitude. However at the end of RUN #2, no option is provided since the program cycles on to the third altitude point in the wind field table, sets up RUN #3 and goes to STOP 3 again for entry of the two balloon parameters at this new altitude. Thus a solution is provided for each altitude in the wind field table except for Z_S. When Z_S is detected, the program will terminate to the OPT. ENT printout. In this case however, a choice of only 0 or 1 will be offered. If 1 is chosen the program will recycle back to the initial maximum altitude and properly retain the complete original wind table.

If 3 is Entered: Hold Cable Length and Change Winds—Use this to find balloon altitude with the cable length determined in a MAX. ALT run held constant and the system reaction to a different wind profile to be entered in Program No. 77.007B. After the 3 is entered, Side 1 of Program No. 77.007B card should be inserted. When CONT is pressed, the reading of the 4 card sides will then proceed. In this case the instructions in Section 3.4.4 should then be followed.

Note No. 1 - If incorrect data is believed to have been entered do <u>not</u> press STOP END to restart program. For correct and safe clearing of registers press following:

STOP

GO TO

1

6

6

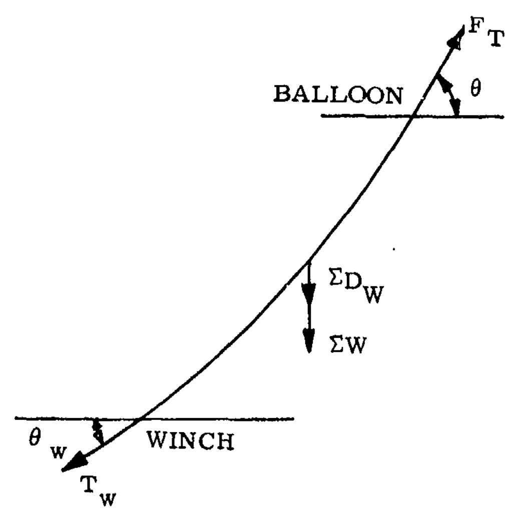
3

After OPT ENT message is printed, enter 0, then:

CONT

Note No. 2 - Proof of solutions can be made by summation of the vertical forces. Summary of the horizontal forces is not possible because summations along two fixed horizontal axes could not be handled within the available space.

Therefore: $F_T \sin\theta = T_W \sin\theta_W + \Sigma W + \Sigma D_W$



Note No. 3 - An ambiguity situation is set up when a no-wind condition at all altitudes is introduced, that is, $\theta = 90°$ at Z_B and wind entries are all 0 knots. In such a case:

(a) The slant range becomes infinity instead of being equal to the height or cable length and,

(b) The terms AZ_B and AZ_C compute as 180° opposite the azimuths used in the wind-field entries. Since the cable elevation angle at the winch is 90° for this condition, these latter terms have no meaning—the balloon is directly over the winch.

3.3.5 INPUT DATA FORM

Test No.:________Date:______________Notes:________________________

INPUT			77.007
STOP NO.	ITEM	VALUE	
1-1-1	MAX. BALLOON ALTITUDE Z_B SURFACE ALTITUDE Z_S CABLE C_D or π for Internal Comp.		Ft. MSL Ft. MSL
2-2-2	CABLE DIAMETER D CABLE WEIGHT per 1000 ft CABLE ELEMENT LENGTH, K OR 0 for $K = (Z_B - Z_S)/100$		Inches Lb. Ft.
3-3-3	BALLOON TOTAL FORCE F_T ANGLE OF TOTAL FORCE θ		Lb. Deg.
4-4-4	WIND PROFILE		

		For Stop 3 Opt. Lower Alt. Cycles			For Stop 3 Opt. Lower Alt. Cycles
1. Z_B Wind AZ	Ft. MSL Knots Deg.		7. Z Wind AZ	Ft. MSL Knots Deg.	F_T Lb. θ Deg.
2. Z Wind AZ	Ft. MSL Knots Deg.	F_T Lb. θ Deg.	8. Z Wind AZ	Ft. MSL Knots Deg.	F_T Lb. θ Deg.
3. Z Wind AZ	Ft. MSL Knots Deg.	F_T Lb. θ Deg.	9. Z Wind AZ	Ft. MSL Knots Deg.	F_T Lb. θ Deg.
4. Z Wind AZ	Ft. MSL Knots Deg	F_T Lb. θ Deg.	10. Z Wind AZ	Ft. MSL Knots Deg.	F_T Lb. θ Deg.
5. Z Wind AZ	Ft. MSL Knots Deg.	F_T Lb. θ Deg.	11. Z Wind AZ	Ft. MSL Knots Deg.	F_T Lb. θ Deg.
6. Z Wind AZ	Ft. MSL Knots Deg.	F_T Lb. θ Deg.	12. Z Wind AZ	Ft. MSL Knots Deg.	

A minimum of two wind points must be specified. Conditions at Z_B must be the first point. Conditions at Z_S must be the last point. A maximum of twelve wind points may be specified.

3.3.6 PROGRAM NO. 77.007 LISTING

STEP	KEY	CODE	X	Y	Z	STEP	KEY	CODE	X	Y	Z
000 0	CLR					0	#				
1	FMT					1	/				
2	FMT					2	CNT				
3	F					3	M				
4	R					4	A				
5	O					5	X				
6	G					6	•				
7	#					7	A				
8	7					8	L				
9	7					9	T				
0	•					0	FMT				
1	0					006 1	/		NW=1		
2	0					2	X→				
3	7					3	b				
4	CLR					4	0		0		
5	3					5	↑			0	
6	–					6	X( )				
7	D					7	0		r	0	
8	I					8	X=Y				
9	M					9	0				
0	•					0	1				
1	T					1	3				
2	E					2	5				
3	T					3	↑			r	
4	H					4	3		3	r	
5	E					5	÷			r/3	
6	R					6	1		1	r/3	
7	CLR					7	+			RUN No	
8	T					8	X( )				
9	E					9	0		r	RUN No	
0	S					0	↑			r	RUN No
1	T					1	3				
2	FMT					2	9		39	r	RUN No
3	STOP		T. No			3	+			3NW+36+r	
4	PNT					4	y→				
5	X( )					5	a				
6	7					6	X( )				
7	7		R_{B1}			7	IND				
8	↑					8	a		Z_B		
9	0		0	R_{B1}		9	X⇌Y			Z_B	RUN No
0	X=Y					0	X( )				
1	FMT					1	4		Z_S	Z_B	RUN No
2	X( )					2	X=Y				
3	CNT					3	1				
4	CNT					4	6				
5	FMT					5	6				
6	FMT					6	3				
7	R					7	y→				
8	U					8	5				
9	N					9	R↑		RUN No	Z_S	Z_B

0100	0	FMT				
	1	FMT				
	2	R				
	3	U				
	4	N				
	5	[illegible]				
	6	FMT				
	7	PNT		R,W X6		
	8	2		2		
	9	X→				
	0	÷				
	1	a				
	2	X( )				
	3	IND				
	4	a		β_B	Z_S	Z_B
	5	R↑		Z_B	β_B	
	6	FMT				
	7	FMT				
	8	B				
	9	.				
	0	A				
	1	L				
	2	T				
	3	+				
	4	A				
	5	Z				
	6	FMT				
	7	PNT		Z_B		
	8	↓		β_B		
	9	PNT				
	0	.PNT				
	1	GoTo				
	2	2				
	3	4				
	4	0				
013	5	X( )				
	6	B		ON		
	7	↑			ON	
	8	0		0	ON	
	9	X=Y				
	0	0				
	1	1				
	2	6				
	3	1				
	4	X( )				
	5	3		Z_B		
	6	X→				
	7	5				
	8	PNT		Z_B		
	9	X( )				

	0	4		Z_S		
	1	PNT		Z_S		
	2	X( )				
	3	1				
	4	2		π or C_D		
	5	PNT				
	6	GoTo				
	7	0				
	8	1				
	9	8				
	0	0				
016	1	1		1		
	2	↑		1	1	
	3	↑		1	1	1
	4	STOP		π or C_D	Z_S	Z_B
	5	R↑		Z_B	π or C_D	Z_S
	6	PNT		Z_B		
	7	X→				
	8	3				
	9	X→				
	0	5				
	1	R↑		Z_S	Z_B	π or C_D
	2	PNT		Z_S		
	3	X→				
	4	4				
	5	R↑		π or C_D	Z_S	Z_B
	6	X→				
	7	1				
	8	2				
	9	PNT		π or C_D		
018	0	2		2		
	1	↑		2	2	
	2	↑		2	2	2
	3	STOP		O or K	Wt	Diam
	4	R↑		D	O or K	Wt
	5	PNT		D		
	6	R↑		Wt	D	O or K
	7	X→				
	8	3				
	9	5				
	0	PNT		Wt		
	1	1				
	2	2		12	D	
	3	÷			D (ft)	O or K
	4	Y→				
	5	1				
	6	1				
	7	↓			O or K	
	8	0		0	O or K	
	9	X<Y				

STEP	KEY	CODE	X	Y	Z
020 0	0				
1	2				
2	1				
3	4				
4	X( )				
5	3		Z_B	0	
6	↑			Z_B	0
7	X( )				
8	4		Z_S	Z_B	0
9	—			$Z_B - Z_S$	
0	1				
1	0				
2	0		100	$Z_B - Z_S$	
3	÷			K	
021 4	↓		K		
5	PNT		K		
6	X→				
7	6				
8	↑			K	
9	X( )				
0	3				
1	5		Wt		
2	↑			Wt	K
3	1				
4	ENT E				
5	3		1000	Wt	K
6	÷			Wt (/ft)	K
7	↓		Wt /ft	K	K
8	X			Wt of K	
9	y→				
0	9				
1	X( )				
2	1				
3	1		D (ft)		K
4	R↑		K	D	
5	X			A	
6	y→				
7	0				
8	1				
9	0				
024 0	3		3		
1	↑		3	3	
2	↑		3	3	3
3	STOP		θ	F_T	
4	X⇄Y		F_T	θ	
5	PNT		F_T		
6	X→				
7	1				
8	X⇄Y		θ	F_T	
9	PNT		θ		

STEP	KEY	CODE	X	Y	Z
0	X→				
1	2				
2	X( )				
3	0		r		
4	↑			r	
5	0		0	r	
6	X<Y				
7	0				
8	4				
9	1				
0	7				
1	FMT				
2	FMT				
3	W				
4	I				
5	N				
6	D				
7	S				
8	FMT				
9	X( )				
0	8		ON		
1	↑			ON	
2	1		1	ON	
3	X=Y				
4	0				
5	3				
6	5				
7	9				
027 8	4		4		
9	↑		4	4	
0	↑		4	4	4
1	STOP		AZ	WIND	Z
2	R↑		Z	AZ	WIND
3	PNT		Z		
4	R↑		WIND	Z	AZ
5	PNT		WIND		
6	R↑		AZ	WIND	Z
7	PNT		AZ		
8	PNT				
9	y→				
0	3				
1	5				
2	X→				
3	3				
4	6				
5	b		NW		
6	X⇄Y			NW	Z
7	3		3	NW	
8	X			3NW	
9	3				

STEP	KEY	CODE	X	Y	Z
030 0	6		36	3NW	Z
1	+			3NW+36	Z
2	y→				
3	a				
4	R↑		Z		
5	X→				
6	IND				
7	a				
8	X()				
9	3				
0	5		W(KNOTS)		
1	X⇄Y			W	
2	1				
3	•				
4	6				
5	8				
6	7				
7	8		1.6878	W,KNOTS	
8	X			V,fps	
9	1		1		
0	X→				
1	+				
2	a				
3	y→				
4	IND				
5	a				
6	X→				
7	+				
8	a				
9	X()				
0	3				
1	6		AZ		
2	X→				
3	IND				
4	a				
5	2		2		
6	X→				
7	−				
8	a				
9	X()				
0	IND				
1	a		Z		
2	↑			Z	
3	X()				
4	4		Zs	Z	
5	X=Y				
6	0				
7	4				
8	1				
9	7				

STEP	KEY	CODE	X	Y	Z
0	1		1		
1	X→				
2	+				
3	b				
4	GoTo				
5	0				
6	2				
7	7				
8	8				
035 9	1		NW=1		
0	X→				
1	b				
2	↑			NW	
3	3		3	NW	
4	X			3NW	
5	3				
6	6		36	3NW	
7	+			3NW+36	
8	y→				
9	a				
037 0	X()				
1	IND				
2	a		Z		
3	PNT		Z		
4	↑			Z	
5	1		1	Z	
6	X→				
7	+				
8	a				
9	X()				
0	IND				
1	a		V	Z	
2	↑			V	Z
3	1				
4	•				
5	6				
6	8				
7	7				
8	8		1.6878	V fps	
9	÷			W KNOTS	
0	↓		WIND	Z	
1	PNT		WIND	Z	
2	1		1		
3	X→				
4	+				
5	a				
6	X()				
7	IND				
8	a		AZ	Z	
9	PNT		AZ		

040	0	PNT				
	1	X( )				
	2	a		[illegible]	[illegible]	
	3	X≠Y				
	4	0				
	5	+				
	6	1				
	7	↓				
	8	1		1	[illegible]	
	9	X→				
	0	+				
	1	a				
	2	GO TO				
	3	0				
	4	3				
	5	7				
	6	0				
041	7	1		NW=1		
	8	X→				
	9	b				
	0	4				
	1	1		41		
	2	↑			41	
	3	X( )				
	4	a		a	41	
	5	+			a+41	
	6	Y→				
	7	a				
	8	X( )				
	9	IND				
	0	a		[illegible]		
	1	X→				
	2	2				
	3	7				
	4	X→				
	5	3				
	6	0				
	7	CNT				
	8	CNT				
	9	CNT				
044	0	X( )				
	1	2		θ		
	2	SIN X		SIN θ		
	3	↑			SIN θ	
	4	X( )				
	5	6		K	SIN θ	
	6	X			√	
	7	X( )				
	8	5		[illegible]	√	
	9	X⇄Y		√	[illegible]	

	0	−		√	[illegible]	
	1	↑			√	[illegible]
	2	X( )				
	3	a		[illegible]	√	[illegible]
	4	R↑			[illegible]	√
	5	X≠Y				
	6	0				
	7	0				
	8	↓				
	9	6				
	0	X( )				
	1	7		K RED	[illegible]	√
	2	X⇄Y		[illegible]	K RED	√
	3	1				
	4	0		10	K RED	√
	5	X=Y				
	6	1				
	7	1				
	8	5				
	9	7				
	0	X→				
	1	7				
	2	X→				
	3	+				
	4	6				
	5	X→				
	6	+				
	7	9				
	8	X→				
	9	+				
	0	1				
	1	0				
	2	GO TO				
	3	4				
	4	4				
	5	0				
048	6	X→		[illegible]'	[illegible]	√
	7	5				
	8	PNT		[illegible]'		
	9	R↑		√	[illegible]'	[illegible]
	0	PNT		√		
	1	X→				
	2	+				
	3	2				
	4	3				
	5	X( )				
	6	2		θ		
	7	COS X		COS θ		
	8	↑			cos θ	
	9	X( )				

Step	Key	Code	X	Y	
05? 0	6		K	cos θ	
1	X→				
2	+				
3	2				
4	4				
5	X			K cos θ	
6	X()				
7	2				
8	8		±A		
9	cos X		cos ±A	K cos θ	
0	X⇄Y		K cos θ	cos ±A	
1	X			h	
2	X⇄Y		h	K cos θ	
3	PNT		h		
4	X→				
5	+				
6	2				
7	1				
8	X()				
9	2				
0	9		±A	K cos θ	
1	sin X		sin ±A	K cos θ	
2	X			Δ'	
3	↓		Δ'		
4	PNT		Δ'		
5	X→				
6	+				
7	2				
8	2				
9	X()				
0	2				
1	3		J		
2	PNT		J		
3	X()				
4	2				
5	1		H		
6	PNT		H		
7	X()				
8	2				
9	2		I		
0	PNT		I		
1	PNT				
2	X()				
3	2				
4	4		±K		
5	PNT		±K		
6	X()				
7	9		W		
8	X→				
9	+				

Step	Key	Code	X	Y	
0	2				
1	8				
2	X()				
3	2				
4	5		±W		
5	PNT		±W'		
05? 6	b		NW		
7	↑			n*	
8	1		1	n'	
9	+		1	n'+1 = NW	
0	3		3	n'	
1	X			3n'	
2	3				
3	6		36	3n'	
4	+			3n'+36	
5	X()				
6	θ		Δ		
7	+			3n'+36+θ	
8	y→				
9	a				
0	X()				
1	IND				
2	a		Z_{n+1}		
3	↑			Z_{n+1}	
4	X()				
5	5		Z	Z_{n+1}	
6	X>Y				
7	0				
8	5				
9	9				
0	0				
1	1		1		
2	X→				
3	+				
4	b				
5	GO TO				
6	2				
7	8				
8	5				
9	6				
059 0	3		3	Z_{n+1}	
1	X→				
2	−				
3	a				
4	X()				
5	IND				
6	a		Z_n	Z_{n+1}	
7	X→				
8	3				
9	5				

*NOTE: FOR CLARITY
n' = NW IN STEP NO 0457–0722

STEP	KEY	CODE	X	Y	Z	STEP	KEY	CODE	X	Y	Z
060 0	X⇄Y		Z_{n+1}	Z_n		0	6				
1	−			ΔZ_W		1	8				
2	1		1			2	7				
3	X→					3	8		1.6878	V_{fps}	
4	+					4	÷			V_{KNOTS}	
5	a					5	↓		V_{KNOTS}		
6	X( )					6	PNT		V_{KNOTS}		
7	IND					7	↑			V	
8	a		V_n	ΔZ_W		8	0		0	V	
9	↑			V_n	ΔZ_W	9	X=Y				
0	3		3			0	1				
1	X→					1	1				
2	÷					2	0				
3	a					3	5				
4	X( )					4	1		1		
5	IND					5	X→				
6	a		V_{n+1}	V_n	ΔZ_W	6	+				
7	X⇄Y		V_n	V_{n+1}		7	a				
8	−		V_n	ΔV	ΔZ_W	8	X( )				
9	X⇄Y		ΔV	V_n	ΔZ_W	9	IND				
0	R↑		ΔZ_W	ΔV	V_n	0	a		AZ_{n+1}		
1	÷		ΔZ_W	dV/dZ		1	↑			AZ_{n+1}	
2	X→					2	3				
3	3					3	X→				
4	6					4	−				
5	X( )					5	a				
6	3					6	X( )				
7	5		Z_n			7	IND				
8	R↑		V_n	Z_n	dV/dZ	8	a		AZ_n	AZ_{n+1}	
9	X→					9	−			ΔAZ_t	
0	3					0	1				
1	5					1	8				
2	X( )					2	0		180	ΔAZ_t	
3	5		Z	Z_n	dV/dZ	3	X<Y				
4	−			ΔZ_o		4	0				
5	y→					5	6				
6	3					6	9				
7	7					7	9				
8	↓		ΔZ_o	dV/dZ		8	CHG S		−180	ΔAZ_t	
9	×			ΔV_o		9	X>Y				
0	X( )					0	0				
1	3					1	7				
2	5		V_n	ΔV_o		2	0				
3	+			V		3	8				
4	y→					4	GoTo				
5	0					5	0				
6	1					6	7				
7	3					7	1				
8	1					8	2				
9	•					069 9	3				

STEP	KEY	CODE	x	y	z
070 0	6				
1	0		360	ΔAZ_t	
2	−			ΔAZ	
3	GoTo				
4	0				
5	7				
6	1				
7	2				
070 8	3			/	
9	6				
0	0		360	ΔAZ_t	
1	+			ΔAZ	
071 2	X()				
3	3				
4	6		ΔZ_W	ΔAZ	
5	÷			dAZ/dZ	
6	X()				
7	3				
8	7		ΔZ_o	dAZ/dZ	
9	X			ΔAZ_o	
0	X()				
1	IND				
2	a		AZ_n	ΔAZ_o	
3	+			AZ_t	
4	3				
5	6				
6	0		360	AZ_t	
7	X<Y				
8	0				
9	7				
0	4				
1	7				
2	0		0	AZ_t	
3	X<Y				
4	0				
5	7				
6	4				
7	8				
8	3				
9	6				
0	0		360	AZ_t	
1	+			AZ	
2	GoTo				
3	0				
4	7				
5	4				
6	8				
074 7	−		360	AZ	
074 8	y→			AZ	
9	1				

STEP	KEY	CODE	x	y	z
0	4				
1	X()				
2	2				
3	7		$\Sigma\beta+\beta_B$	AZ	
4	X⇄Y		AZ	$\Sigma\beta+\beta_B$	
5	PNT		AZ		
6	X⇄Y		$\Sigma\beta+\beta_B$	AZ	
7	PNT		$\Sigma\beta+\beta_B$		
8	−			α_t	
9	0		0	α_t	
0	X>Y				
1	0				
2	7				
3	7				
4	0				
5	GoTo				
6	0				
7	7				
8	7				
9	4				
0770	3				
1	6				
2	0		360	α_t	
3	+			α	
0774	y→				
5	1				
6	5				
7	↓		α		
8	PNT		α		
9	cos X		$\cos\alpha$		
0	X^2		$\cos^2\alpha$		
1	↑			$\cos^2\alpha$	
2	X()				
3	2		θ		
4	cos X		$\cos\theta$		
5	X^2		$\cos^2\theta$	$\cos^2\alpha$	
6	X			$\cos^2\cos^2$	
7	1		1		
8	X⇄Y		$\cos^2\cos^2$	1	
9	−			$\cos^2\phi$	
0	↓		$\cos^2\phi$		
1	$\sqrt{X}$		$\cos\phi$		
2	X→				
3	2				
4	9				
5	↑			$\cos\phi$	
6	X()				
7	1				
8	3		V	$\cos\phi$	
9	X			$\sqrt{N}$	

STEP	KEY	CODE	X	Y	Z	STEP	KEY	CODE	X	Y	Z
080 0	y→			V_N		0	CHG S		1.2^{-5}	$6.8^{-11}Z$	Z
1	1					1	X⇄Y		$6.8^{-11}Z$	1.2^{-5}	
2	6					2	−			μ	Z
3	X( )					3	↓		μ	Z	Z
4	5		Z			085 4	X→				
5	↑		Z	Z		5	3				
6	↑		Z	Z	Z	6	3				
7	3					7	X( )				
8	6					8	7				
9	5					9	5		a_1	Z	
0	0					0	X			a_1Z	
1	0		36500	Z	Z	1	X( )				
2	X>Y					2	7				
3	0					3	6		a_0	a_1Z	
4	8					4	+			a_0+a_1Z	Z
5	3					5	↓		a_0+a_1Z	Z	Z
6	1					6	X			$\ln \rho/\rho_0$	
7	•					7	↓		$\ln \rho/\rho_0$		
8	9					8	e^x		ρ/ρ_0		
9	5					9	↑			ρ/ρ_0	
0	5					0	•				
1	2					1	0				
2	8					2	0				
3	ENT E					3	2				
4	5					4	3				
5	CHG S		μ	Z	Z	5	7				
6	GoTo					6	8		ρ_0	ρ/ρ_0	
7	0					7	X			$\rho\left(\frac{slug}{ft^3}\right)$	
8	8					8	y→				
9	5					9	0				
0	4					0	3				
083 1	6					1	2				
2	•					2	3				
3	8					3	2				
4	4					4	•				
5	1					5	1				
6	6					6	7				
7	4					7	4		32.174	ρ	
8	ENT E					8	X			$\rho\left(\frac{lbs}{ft^3}\right)$	
9	1					9	X( )				
0	1					0	1				
1	CHG S		6.8^{-11}	Z	Z	1	6		V_N	ρ	
2	X			$6.8^{-11}Z$	Z	2	X			ρV_N	
3	1					3	X( )				
4	•					4	1				
5	2					5	1		D	ρV_N	
6	0					6	X			$\rho V_N D$	
7	5					7	X( )				
8	ENT E					8	3				
9	5					9	3		μ	$\rho V_N D$	

STEP	KEY	CODE	X	Y	Z	STEP	KEY	CODE	X	Y	Z
090 0	÷			R		0	X<Y				
1	↓		R			1	0				
2	PNT		R			2	9				
3	↑			R		3	6				
4	X( )					4	1				
5	1					5	1		1 = n_{CD}	R	1
6	2		π or C_D	R		6	GoTo				
7	↑			π or C_D	R	7	1				
8	π		π	π or C_D	R	8	0				
9	X=Y					9	4				
0	0					0	8				
1	9					096 1	9				
2	1					2	0				
3	9					3	0		900	R	
4	GoTo					4	X<Y				
5	1					5	0				
6	0					6	9				
7	8					7	7				
8	3					8	5				
091 9	1		1	π	R	9	2		2 = n_{CD}	R	
0	R↑		R	1	π	0	GoTo				
1	X>Y					1	1				
2	0					2	0				
3	9					3	4				
4	4					4	8				
5	8					097 5	4				
6	↑			R	1	6	5				
7	1					7	0				
8	0					8	0		4500	R	
9	•					9	X<Y				
0	9		10.9	R	1	0	0				
1	X⇄Y		R	10.9		1	9				
2	÷		R	10.9/R		2	9				
3	TAB					3	0				
4	4		LOG R	10.9 R		4	3		3 = n_{CD}	R	
5	↑			LOG R	10.9/R	5	GoTo				
6	•					6	1				
7	8					7	0				
8	7		.87	LOG R	10.9/R	8	4				
9	X⇄Y		LOG R	.87		9	8				
0	−			.87−LOG R	10.9/R	099 0	9				
1	↓		.87−LOG R	10.9/R		1	ENTE				
2	÷			C_D		2	3		9000	R	
3	GoTo					3	X<Y				
4	1					4	1				
5	0					5	0				
6	8					6	0				
7	3					7	4				
094 8	↑			R	1	8	4		4 = n_{CD}	R	
9	9		9	R	1	9	GoTo				

STEP	KEY	CODE	x	y	z	STEP	KEY	CODE	x	y	z
100 0	1					0	X			$3n_{CD}$	R
1	0					1	7				
2	4					2	4		74	$3n_{CD}$	
3	8					3	+			$3n_{CD}+74$	R
100 4	4					4	y→				
5	ENTE					5	a				
6	4		40000	R		6	X( )				
7	X<Y					7	IND				
8	1					8	a		R_B		
9	0					9	TAB				
0	1					0	4		$LOG\ R_B$		
1	8					1	X⇄Y			$LOG\ R_B$	R
2	5		$5=n_{CD}$	R		2	2		2		
3	GoTo					3	X→				
4	1					4	+				
5	0					5	a				
6	4					6	R↑		R		$LOG\ R_B$
7	8					7	TAB				
101 8	5					8	4		$LOG\ R$		$LOG\ R_B$
9	ENTE					9	R↓		$LOG\ R_B$	$LOG\ R$	
0	4		50000	R		0	−			$LR-LR_B$	
1	X<Y					1	X( )				
2	1					2	IND				
3	0					3	a		K_B		
4	3					4	X			$K_B(\ )$	
5	2					5	1		1		
6	6		$6=n_{CD}$	R		6	X→				
7	GoTo					7	−				
8	1					8	a				
9	0					9	X( )				
0	4					0	IND				
1	8					1	a		C_{DB}	$K_B(\ )$	
103 2	2					2	+			C_D	
3	5					108 3	↓		C_D		
4	ENTE					4	PNT		C_D		
5	4		250 000	R		5	↑			C_D	
6	X<Y					6	X( )				
7	1					7	1				
8	0					8	6		V_N	C_D	
9	4					9	X^2		V_N^2	C_D	
0	7					0	↑			V_N^2	C_D
1	7		$7=n_{CD}$	R		1	X( )				
2	GoTo					2	3				
3	1					3	2		ρ	V_N^2	C_D
4	0					4	X			ρV_N^2	C_D
5	4					5	2		2	ρV_N^2	C_D
6	8					6	÷			q	
104 7	8		$8=n_{CD}$	R		7	↓		q	C_D	
104 8	↑			n_{CD}	R	8	PNT		q		
9	3		3	n_{CD}	R	9	X			qC_D	

STEP	KEY	CODE	X	Y	Z
110 0	X( )				
1	1				
2	0		A	qC_D	
3	×			D_T	
4	↓		D_T		
110 5	X→				
6	3				
7	1				
8	PNT		D_T		
9	↑			D_T	
0	X( )				
1	2		θ	D_T	
2	↑			θ	D_T
3	9				
4	0		90	θ	D_T
5	X>Y				
6	1				
7	1				
8	3				
9	1				
0	0		0	θ	D_T
1	R↑		D_T	0	θ
2	X⇄Y		0	D_T	θ
3	X=Y				
4	1				
5	2				
6	9				
7	1				
8	↓		D_T	θ	
9	X⇄Y		θ	D_T	
0	↑			θ	D_T
113 1	↓		θ	D_T	
2	cos X		$\cos\theta$	D_T	
3	×			$D_T\cos\theta$	
4	X( )				
5	1				
6	5		α		
7	cos X		$\cos\alpha$	$D_T\cos\theta$	
8	×		$\cos\alpha$	D_W	
9	X⇄Y		D_W	$\cos\alpha$	
0	X→				
1	1				
2	7				
3	X→				
4	+				
5	2				
6	6				
7	PNT		D_W		
8	X( )				
9	2				

STEP	KEY	CODE	X	Y	Z
0	9		$\cos\phi$	$\cos\alpha$	
1	×			$\cos\phi\cos\alpha$	
2	↑			$\cos\phi$	$\cos\phi\cos\alpha$
3	X( )				
4	3				
5	1		D_T	$\cos\phi$	$\cos\phi\cos\alpha$
6	R↑		$\cos\phi\cos\alpha$	D_T	$\cos\phi$
7	×			D_H	
8	↓		D_H	$\cos\phi$	
9	PNT		D_H		
0	X→				
1	1				
2	8				
3	X( )				
4	3				
5	1		D_T	$\cos\phi$	
6	×			$D_T\cos\phi$	
7	X( )				
8	1				
9	5		α		
0	SIN X		$\sin\alpha$	$D_T\cos\phi$	
1	×			D_S	
2	↓		D_S		
3	PNT		D_S		
4	PNT				
5	CNT				
6	CNT				
7	CNT				
8	↑			D_S	
9	X( )				
0	2		θ		
1	COS X		$\cos\theta$	D_S	
2	↑			$\cos\theta$	D_S
3	X( )				
4	1		F_T or T	$\cos\theta$	
5	×			$T\cos\theta$	D_S
6	X( )				
7	1				
8	8		D_H		
9	+			$T\cos\theta+D_H$	D_S
0	↓		$T\cos\theta+D_H$	D_S	D_S
1	X→				
2	2				
3	0				
4	÷			$\tan\beta_1$	D_S
5	↓		$\tan\beta_1$		
6	arc				
7	tan X		β_1		
8	GO TO				
9	LABEL				

STEP	KEY	CODE	X	Y	Z
120 0	π				
120 1	PNT		β_1		
2	X→				
3	+				
4	2				
5	7				
6	X→				
7	+				
8	2				
9	8				
0	cos X		cos β_1		
1	↑			cos β_1	
2	X( )				
3	2		θ	cos β_1	
4	SIN X		SIN θ	cos β_1	
5	↑			SIN θ	cos β_1
6	X( )				
7	1		F_T OR T	SIN θ	cos β_1
8	×			T SIN θ	
9	X( )				
0	9		W		
1	−			TSIN − W	
2	X( )				
3	1				
4	7		DW		
5	−			TSIN−W−DW	
6	X( )				
7	2				
8	0		Tcos+D_H		
9	÷			( )/( )	cos β_1
0	↓		( )/( )	cos β_1	
1	×			tan θ_1	
2	↓		tan θ_1	cos β_1	
3	arc				
4	tan X		θ_1	cos β_1	
5	PNT		θ_1		
6	X→				
7	2				
8	cos X		cos θ_1	cos β_1	
9	×			cos θ_1 cos β_1	
0	X( )				
1	2				
2	0		Tcos+D_H		
3	X⇄Y		cosθ cosβ	Tcos+D_H	
4	÷			T_1	
5	↓		T_1		
6	X→				
7	1				
8	PNT		T_1		
9	PNT				

STEP	KEY	CODE	X	Y	Z
0	X( )				
1	2				
2	7		$\Sigma\beta+\beta_B$		
3	↑			$\Sigma\beta+\beta_B$	
4	3				
5	6				
6	0		360	$\Sigma\beta+\beta_B$	
7	X<Y				
8	1				
9	2				
0	7				
1	3				
2	0		0	$\Sigma\beta+\beta_B$	
3	X>Y				
4	1				
5	2				
6	7				
7	9				
8	GoTo				
9	1				
0	2				
1	8				
2	3				
127 3	−		360	$\Sigma\beta+\beta_B$	
4	GoTo				
5	1				
6	2				
7	8				
8	3				
127 9	3				
0	6				
1	0		360		
2	+		360	$\Sigma\beta+\beta_B$	
128 3	y→				
4	2				
5	7				
6	GoTo				
7	1				
8	3				
9	0				
0	5				
129 1	X( )				
2	2		$\theta_1 = 90$		
3	PNT				
4	X( )				
5	1		F_T or T		
6	↑			T	
7	X( )				
8	9		W	T	
9	−			T_1	

STEP	KEY	CODE	X	Y	Z
130 0	y→			T_1	
1	1				
2	↓		T_1		
3	PNT		T_1		
4	PNT				
130 5	X( )				
6	1		T_1		
7	↑			T_1	
8	0		0	T_1	
9	X<Y				
0	1				
1	3				
2	3				
3	1				
4	FMT				
5	FMT				
6	T				
7	E				
8	N				
9	S				
0	I				
1	O				
2	N				
3	•				
4	O				
5	FMT				
6	GoTo				
7	1				
8	3				
9	7				
0	0				
133 1	X( )				
2	2		θ_1		
3	↑			θ_1	
4	0		0	θ_1	
5	X<Y				
6	0				
7	4				
8	3				
9	7				
0	FMT				
1	FMT				
2	C				
3	A				
4	B				
5	L				
6	E				
7	CNT				
8	H				
9	O				

STEP	KEY	CODE	X	Y	Z
0	R				
1	FMT				
2	GoTo				
3	1				
4	3				
5	7				
6	0				
.135 7	FMT				
8	FMT				
9	O				
0	N				
1	CNT				
2	S				
3	U				
4	R				
5	F				
6	A				
7	C				
8	E				
9	FMT				
137 0	FMT				
1	FMT				
2	A				
3	L				
4	T				
5	FMT				
6	X( )				
7	5		Z		
8	PNT		Z		
9	FMT				
0	FMT				
1	H				
2	T				
3	FMT				
4	X( )				
5	2				
6	3		J		
7	PNT		J		
8	FMT				
9	FMT				
0	T				
1	E				
2	N				
3	S				
4	I				
5	O				
6	N				
7	FMT				
8	X( )				
9	1		T		

STEP	KEY	CODE	X	Y	Z	STEP	KEY	CODE	X	Y	Z
140 0	PNT					0	6		ΣD_W		
1	FMT					1	PNT		ΣD_W		
2	FMT					2	FMT				
3	C					3	FMT				
4	•					4	H				
5	E					5	,				
6	L					6	I				
7	E					7	,				
8	V					8	L				
9	•					9	FMT				
0	A					0	X( )				
1	N					1	2				
2	G					2	1		H		
3	FMT					3	PNT		H		
4	X( )					4	↑			H	
5	2		θ			5	X( )				
6	PNT		θ			6	2				
7	FMT					7	2		I	H	
8	FMT					8	PNT		I		
9	L					9	To POL		L	δ	
0	E					0	X→				
1	N					1	3				
2	G					2	5				
3	T					3	PNT		L		
4	H					4	X( )				
5	FMT					5	3				
6	X( )					6	0		β_B	δ	
7	2					7	↑			β_B	δ
8	4		ΣK			8	9				
9	PNT		ΣK			9	0		90	β_B	δ
0	FMT					0	−			$\beta_B - 90$	
1	FMT					1	↓		$\beta_B - 90$	δ	
2	W					2	X⇄Y		δ	$\beta_B - 90$	
3	T					3	−			AZ_B	
4	FMT					4	3				
5	X( )					5	6				
6	2		ΣW			6	0		360	AZ_B	
7	5					7	X<Y				
8	PNT		ΣW			8	1				
9	FMT					9	5				
0	FMT					0	0				
1	V					1	3				
2	•					2	0		0	AZ_B	
3	D					3	X>Y				
4	R					4	1				
5	A					5	5				
6	G					6	0				
7	FMT					7	9				
8	X( )					8	Go To				
9	2					9	1				

Step	Key	Code	X	Y	Z
150 0	3				
1	/				
2	3				
150 3	—		360	AZ_B	
4	GoTo				
5	/				
6	5				
7	/				
8	3				
150 9	3				
0	6				
1	0		360	AZ_B	
2	+			AZ_B	
151 3	↓		AZ_B		
4	FMT				
5	FMT				
6	A				
7	Z				
8	•				
9	T				
0	O				
1	CNT				
2	B				
3	L				
4	N				
5	FMT				
6	PNT		AZ_B		
7	X→				
8	3				
9	6				
0	X( )				
1	2				
2	3		J		
3	↑			J	
4	X( )				
5	3				
6	5		L	J	
7	÷		L	TAN ε	
8	X⇄Y		TAN ε	L	
9	ARC				
0	TAN X		ε	L	
1	FMT				
2	FMT				
3	B				
4	•				
5	E				
6	L				
7	•				
8	A				
9	N				

Step	Key	Code	X	Y	Z
0	G				
1	FMT				
2	PNT		ε		
3	COS X		COS ε	L	
4	÷			S.R	
5	↓		S.R		
6	FMT				
7	FMT				
8	S				
9	•				
0	R				
1	FMT				
2	PNT		S.R		
3	X( )				
4	2				
5	7		$\Sigma B+B_B$		
6	↑				
7	1				
8	8				
9	0		180	$\Sigma B+B_B$	
0	—			AZ_C	
1	0		0	AZ_C	
2	X>Y				
3	1				
4	5				
5	8				
6	2				
7	GoTo				
8	1				
9	5				
0	8				
1	6				
158 2	3				
3	6				
4	0		360	AZ_C	
5	+			AZ_C	
158 6	3				
7	6				
8	0		360	AZ_C	
9	X>Y				
0	1				
1	5				
2	9				
3	5				
4	—		360	AZ_C	
159 5	↓		AZ_C		
6	FMT				
7	FMT				
8	C				
9	A				

STEP	KEY	CODE	X	Y	Z	STEP	KEY	CODE	X	Y	Z
1600	B					0	+				
1	L					1	N				
2	E					2	FMT				
3	CNT					3	PNT		Y		
4	A					4	X( )				
5	Z					5	0		h		
6	FMT					6	↑			h	
7	PNT		AZ_C			7	0		0	h	
8	X( )					8	X≤Y				
9	3					9	1				
0	6		AZ_B			0	7				
1	↑			AZ_B		1	9				
2	9					2	3				
3	0		90	AZ_B		1663	FMT				
4	X>Y					4	FMT				
5	1					5	O				
6	6					6	P				
7	2					7	T				
8	2					8	•				
9	4					9	E				
0	5					0	N				
1	0		450	AZ_B		1	T				
1622	X⇄Y		AZ_B	450 OR 90		2	CLR				
3	—			γ		3	O				
4	↓		γ			4	—				
5	↑		γ	γ		5	N				
6	SIN X		SIN γ	γ		6	E				
7	X⇄Y		γ	SIN γ		7	W				
8	COS X		COS γ	SIN γ		8	CNT				
9	↑			COS γ	SIN γ	9	P				
0	X( )					0	R				
1	3					1	O				
2	5		L	COS γ	SIN γ	2	B				
3	X		L	X	SIN γ	3	CLR				
4	R↑		SIN γ	L	X	4	1				
5	X			Y	X	5	—				
6	R↑		X		Y	6	N				
7	FMT					7	E				
8	FMT					8	W				
9	X					9	CNT				
0	CNT					0	—				
1	+					1	S				
2	E					2	A				
3	FMT					3	M				
4	PNT		X			4	E				
5	R↑		Y	X		5	CNT				
6	FMT					6	A				
7	FMT					7	L				
8	Y					8	T				
9	CNT					9	S				

STEP	KEY	CODE	X	Y	Z	STEP	KEY	CODE	X	Y	Z
170 0	CNT					0	CNT				
1	CNT					1	C				
2	/					2	H				
3	W					3	G				
4	I					4	•				
5	N					5	W				
6	D					6	I				
7	S					7	N				
8	FMT					8	D				
9	X( )					9	S				
0	0		r			0	*				
1	↑			r		1	H				
2	0		0	r		2	A				
3	X<Y					3	V				
4	1					4	E				
5	7					5	CNT				
6	8					6	CNT				
7	9					7	7				
8	FMT					8	7				
9	FMT					9	•				
0	2					0	0				
1	—					1	0				
2	L					2	7				
3	O					3	B				
4	W					4	CNT				
5	CNT					5	C				
6	C					6	A				
7	Y					7	R				
8	C					8	D				
9	L					9	S				
0	E					0	CLR				
1	S					1	CNT				
2	CLR					2	CNT				
3	3					3	R				
4	—					4	E				
5	H					5	A				
6	O					6	D				
7	L					7	Y				
8	D					8	FMT				
9	CNT					178 9	STOP		ON		
0	C					0	PNT				
1	•					1	X→				
2	L					2	8				
3	E					179 3	CNT			r	
4	N					4	3		3	r	
5	G					5	+			r'	
6	T					6	y→				
7	H					7	0				
8	CLR					8	X( )				
9	CNT					9	7		K RED		

180	0	↑			K Reg
	1	1			
	2	0		10	K Reg
	3	X>Y			
	4	1			
	5	8			
	6	1			
	7	9			
	8	X→		10	
	9	X			
	0	6			
	1	X→			
	2	X			
	3	9			
	4	X→			
	5	X			
	6	0			
	7	1			
	8	0			
181	9	0		0	
	0	X→			
	1	7			
	2	↑			0
	3	X( )			
	4	8		ON	0
	5	X=Y			
	6	1			
	7	8			
	8	9			
	9	6			
	0	X( )			
	1	2			
	2	4		±K	0
	3	X→			
	4	1			
	5	0			
	6	2			
	7	2			0
	8	1		21	0
	9	X→			
	0	a			
184	1	y→			
	2	IND			
	3	a			
	4	1		1	0
	5	X→			
	6	+			
	7	a			
	8	2			
	9	9		29	0

	0	↑			29	0
	1	a		[illegible]	29	0
	2	X=Y				
	3	1				
	4	8				
	5	6				
	6	9				
	7	↓		29	0	
	8	GoTo				
	9	1				
	0	8				
	1	4				
	2	1				
186	3	X( )				
	4	8		ON		
	5	↑			ON	
	6	2		2	ON	
	7	X=Y				
	8	0				
	9	0				
	0	6				
	1	1				
	2	0		0.1		
	3	X→				
	4	0				
	5	X( )				
	6	8		ON		
	7	↑			ON	
	8	3		3	ON	
	9	X=Y				
	0	1				
	1	8				
	2	8				
	3	6				
	4	GoTo				
	5	0				
188	6	CNT				
	7	1		1		
	8	X→				
	9	b				
	0	10				
	1	↑				
	2	↑				
	3	FMT				
	4	GoTo				
	5	CNT				
189	6	0		0		
	7	X→				
	8	a				
189	9	X→				

190	IND				
1	[illegible]				
2	/		1		
3	X				
4	√				
5	[illegible]				
6	7				
7	*		75		
8	[illegible]		[illegible]	75/75	
9	X→Y				
0	0				
1	0				
2	0				
3	0				
4	0		0		
5	[illegible]				
6	/				
7	0				
8	0				
9	[illegible]				
0	/				
1	*			A1	
2	X( )				
3	*		0	A1	
4	*			0	A1
5	7				
6	[illegible]		75	0	A1
7	X→Y				
8	/				
9	/				
0	[illegible]				
1	X( )				
2	2				
3	0		[illegible]	0	A1
4	XDU		A1	[illegible]	
5	0		0	[illegible]	
6	X→Y				
7	/				
8	[illegible]				
9	0				
0	/				
1	0		180		A1
2	X↑		A1	180	
3	*			A1	
4	*				A1

195	KA		A1		
1	[illegible]				
2	/				
3	1				
4	0				
5	/				
6	[illegible]				

Storage Registers

STORAGE		
b	NW WIND CODE	
a	IND. ADRES.	
000	r CYCLE CODE	
001	$F_T \rightarrow T$	
002	Θ	
003	Z_R	
004	Z_S	
005	Z	
006	K	
007	K Red. 0/10	
008	ON OPTION CODE	
009	W	
010	A	
011	D	
012	π or C_D	
013	V	
014	AZ	
015	OC	
016	V_N	
017	D_W	
018	D_H	
019		
020	$7\cos\theta + D_H$	
021	H	SUMMARIES
022	I	SUMMARIES
023	J	SUMMARIES
024	ΣK	SUMMARIES
025	ΣW	SUMMARIES
026	ΣDW	SUMMARIES
027	$\Sigma \beta + \beta_B$	SUMMARIES
028	$\Sigma \beta$	SUMMARIES
029	$\cos\phi$	
030	β_B	
031	D_T	
032	ρ	
033	μ	
034	--	
035	Temp	
036	Temp	
037	Temp	
038	Temp	
039	Z_1	WIND FIELD

040	V_1	WIND FIELD
041	AZ_1	WIND FIELD
042	Z_2	WIND FIELD
043	V_2	WIND FIELD
044	AZ_2	WIND FIELD
045	Z_3	WIND FIELD
046	V_3	WIND FIELD
047	AZ_3	WIND FIELD
048	Z_4	WIND FIELD
049	V_4	WIND FIELD
050	AZ_4	WIND FIELD
051	Z_5	WIND FIELD
052	V_5	WIND FIELD
053	AZ_5	WIND FIELD
054	Z_6	WIND FIELD
055	V_6	WIND FIELD
056	AZ_6	WIND FIELD
057	Z_7	WIND FIELD
058	V_7	WIND FIELD
059	AZ_7	WIND FIELD
060	Z_8	WIND FIELD
061	V_8	WIND FIELD
062	AZ_8	WIND FIELD
063	Z_9	WIND FIELD
064	V_9	WIND FIELD
065	AZ_9	WIND FIELD
066	Z_{10}	WIND FIELD
067	V_{10}	WIND FIELD
068	AZ_{10}	WIND FIELD
069	Z_{11}	WIND FIELD
070	V_{11}	WIND FIELD
071	AZ_{11}	WIND FIELD
072	Z_{12}	WIND FIELD
073	V_{12}	WIND FIELD
074	AZ_{12}	WIND FIELD
075	a_1	
076	a_0	
077	R_B	1
078	C_{DB}	1
079	K_R	1

080	R_B	2
081	C_{DB}	2
082	K_R	2
083	R_B	3
084	C_{DB}	3
085	K_R	3
086	R_B	4
087	C_{DB}	4
088	K_R	4
089	R_B	5
090	C_{DB}	5
091	K_R	5
092	R_B	6
093	C_{DB}	6
094	K_R	6
095	R_B	7
096	C_{DB}	7
097	K_R	7
098	R_B	8
099	C_{DB}	8
100	K_R	8
101	ΣK	
102		
103		
104		
105		
106		
107		
108		

LOADED FROM DATA CARD

3.3.7 SAMPLE INPUT/OUT PRINT

The following are copies of the HP printed tape for Test No. 3 in Section 4.

A. No Intermediate
   Altitude Print

```
PROG#77.007
3-DIM.TETHER
TEST
            3.000*
RUN#1 MAX.ALT
       14000.000
        4000.000
           3.142
           0.280
          25.000
         200.000
        3200.000
          85.000
WINDS
       14000.000
          25.000
         180.000

       13000.000
          25.000
         225.000

       11000.000
          35.000
         270.000

       10000.000
          40.000
         300.000

        8500.000
          30.000
         270.000

        4000.000
          15.000
         210.000
```

```
ON SURFACE
ALT
        4019.508
HT
        9980.492
TENSION
        2951.693
C.ELEV.ANG
          78.986
LENGTH
       10080.000
WT
         252.000
V.DRAG
          38.498
H,I,L
        1005.997
         821.072
        1298.533
AZ.TO BLN
          39.221
B.EL.ANG
          82.587
S.R
       10064.611
CABLE AZ
          53.198
X +E
         821.072
Y +N
        1005.997
OPT.ENT
0-NEW PROB
1-NEW -SAME ALTS
  /WINDS
2-LOW CYCLES
3-HOLD C.LENGTH
  CHG.WINDS*HAVE
  77.007B CARDS
  READY
           2.000*
```

77.007

RUN#	2.000	3.000	5.000
B.ALT/AZ	13000.000 225.000	11000.000 270.000	8500.000 270.000
	3200.000 85.000	3200.000 85.000	3200.000 85.000
ON SURFACE			
ALT	4011.806	4015.719	4010.652
HT	8988.194	6984.281	4489.348
TENSION	2976.535	3026.348	3087.994
C.ELEV.ANG	77.003	78.375	82.274
LENGTH	9120.000	7080.000	4520.000
WT	228.000	177.000	113.000
V.DRAG	59.538	46.553	14.858
H,I,L	1370.465 538.727 1472.550	1130.483 75.843 1133.025	519.743 -36.915 521.052
AZ.TO BLN	66.460	93.838	85.937
B.EL.ANG	80.696	80.785	83.380
S.R	9108.020	7075.587	4519.485
CABLE AZ	70.065	89.889	81.568
X +E	1350.003	1130.483	519.743
Y +N	588.128	-75.843	36.915

OPT.ENT
0-NEW PROB
1-NEW -SAME ALTS
/WINDS

B. Full Print of All Intermediate Altitude Data

```
PROG#77.007
3-DIM.TETHER
TEST
          3.000*
RUN#1 MAX.ALT
      14000.000
       4000.000
          3.142
          0.280
         25.000
        500.000
       3200.000
         85.000
WINDS
      14000.000
         25.000
        180.000

      13000.000
         25.000
        225.000

      11000.000
         35.000
        270.000

      10000.000
         40.000
        300.000

       8500.000
         30.000
        270.000

       4000.000
         15.000
        210.000
```

13501.903	- Z, Altitude, Bottom First Element
498.097	- j
43.578	- h
0.000	- i
498.097	- J
43.578	- H
0.000	- I
500.000	- ΣK, Cable Length
12.500	- ΣW, Cable Weight
25.000	- Wind, knots
202.414	- Azimuth of Wind
180.000	- $\Sigma\beta + \beta_B$
22.414	- α
4468.246	- R, Reynolds Number
0.980	- C_D
1.393	- q, Dynamic Pressure
15.922	- D_T, Total Element Drag
1.283	- D_W, Vert. Drag Component
14.671	- D_H
6.051	- D_S
1.181	- β_1
84.715	- θ_1
3187.593	- T_1
13004.029	- Start of next element printout
497.874	
46.049	
0.949	
995.971	
89.627	
0.949	
1000.000	
25.000	
25.000	
224.819	
181.191	
43.638	
4532.719	
0.982	
1.418	
16.256	
1.084	
11.738	
11.193	
2.099	
84.477	
3175.195	

```
4095.816                          
 491.216
  54.894
  75.456
9904.184
 976.577
 806.570

10000.000
  250.000
   15.319
  211.278
  233.964
  337.313
 3432.494
    0.980
    0.685
    7.835
    1.349
    7.121
   -2.977

   -0.204
   79.058
 2955.245

 4046.725
   49.091
    5.624
    7.645
 9953.275
  982.201
  814.215

10050.000
  251.250
   15.156
  210.623
  233.660
  336.963
 3398.281
    0.980
    0.671
    0.767
    0.134
    0.695
   -0.296

   -0.030
   79.040
 2954.018
```

```
ON SURFACE
ALT
       4046.725
HT
       9953.275
TENSION
       2954.018
C.ELEV.ANG
         79.040
LENGTH
      10050.000
WT
        251.250
V.DRAG
         36.435
H,I,L
        982.201
        814.215
       1275.799
AZ.TO BLN
         39.658
B.EL.ANG
         82.696
S.R
      10034.707
CABLE AZ
         53.630
X +E
        814.215
Y +N
        982.201
OPT.ENT
0-NEW PROB
1-NEW -SAME ALTS
  /WINDS
2-LOW CYCLES
3-HOLD C.LENGTH
  CHG.WINDS*HAVE
  77.007B CARDS
  READY
```

In the printouts above, Case A shows the input data for a balloon at 14,000 ft, left side of first page, followed immediately with the surface or winch data on the right side of the page. At the end, an Option 2 or lower altitude cycle rerun mode was selected from 4 choices provided. On the next page, Runs 2, 3, and 5 are shown. Run 4 is not shown. Run 2, for example, indicates the balloon is at 13,000 ft with a wind azimuth of 225°. The same values of F_T and θ as in Run 1 were entered — an approximation since F_T and θ change with altitude and wind speed. Computations then commence leading directly to the printing of the surface or winch conditions. Since an altitude of 8500 ft is the last altitude above the surface in the wind field table (Run 1), the Run 5 for this altitude is the last of the possible rerun cycles. The program then terminates with a shorter option rerun message, 0 or 1.

On the next page, Case B, the same balloon problem is used in the basic form of the program where all parameters are printed at intermediate altitudes between the balloon and the surface. The altitudes are determined by the location of the bottom-end-point of each cable element and are therefore a function of the element length selected and the elevation angle of each element. On the right side of the page, the parameters for an altitude of 13,501.903 ft, the bottom of the first element, are shown followed by 13,004.029 ft, the second element. On the next page, the last two intermediate altitudes are shown followed by the final surface or winch printout.

It may be noted that while the same balloon, cable, altitudes, and winds were specified in the two cases, the element length, K, was 200 ft in Case A and 500 ft in Case B. As a result, the intercept surface altitudes differ by about 27 ft for these two specific cases. Because of this and the different averaging over the whole altitude range, the other surface parameters also differ by small amounts as will be discussed in Section 4.1.

3.4 Program No. 77.007B

This program, unlike 77.007 and 77.007P, solves a case where the cable length is known and held fixed, while various wind fields are introduced and the balloon permitted to rise or fall to a different equilibrium altitude.

3.4.1 SPECIAL NOTES FOR OPERATION

As indicated in Table 2, this program is entered only after a solution is found using 77.007 for a fixed balloon altitude, cable, and wind profile. Entry of cards for 77.007B is called automatically whenever OPT. ENT-3 is selected at the end of a problem solution in 77.007 (see Section 3.3.2).

With Program 77.007B entered, the storage registers containing the needed parameters from the Program 77.007 solution are left intact. The mathematics and procedures are only slight variations of those used in Programs 77.007 and 77.007P.

A complete solution to the surface is made which yields a cable length. An incremental altitude (ΔZ = 200 ft is built-in but may be changed in STEPS 0044, 0045, 0046, and 0047) increase is then made followed by the solution to a second cable length. These cable lengths are compared to determine if they are progressing in the proper "direction" towards the original cable length. Continued altitude increments, either up or down, are made until the cable length solution nearly equals the original value. A vernier solution with ΔZ = 20 ft similar to the K/10 procedures is then made for more precise conditions. This logic had to be used since unlike a two-dimensional wind field, an increase in wind magnitude for example, does not always result in a lower balloon. Wind azimuth changes with higher wind velocities on the cable could produce a higher balloon condition.

The balloon total force, F_T and its angle, θ, are needed as inputs into this program. However, the balloon altitude and the wind at that altitude are unknown until a solution is found. Therefore, an estimate of F_T and θ must be made for the, as yet, unknown balloon altitude. Some repeat runs may be found necessary before the balloon altitude, the wind at that altitude, and the F_T and θ values are all compatible. As experience is gained with one particular balloon and cable, these estimations will become more exact. Repeated runs with Programs Nos. 76.003 or 76.005 for a particular balloon at various altitudes and wind magnitudes can be used to produce a chart of F_T and θ for any particular balloon as an aid in their selection. Safe balloon operation would dictate that in the original solution under Program 77.007 either; (a) that the altitude selected be the highest allowable—envelope full, balloonet empty—and that any wind changes permissable hereunder would only lower the balloon, or (b) that the proper ballonet-fullness effects be included in the original F_T and θ used with the problem in 77.007.

The wind profile is entered in the same manner as in Programs 77.007 or 77.007P except for one additional requirement. Because of the logic used, one

positive increment always moves the balloon up in altitude from its original value regardless of whether the eventual solution shows the balloon higher or lower. Therefore, the first wind entry must be for an altitude higher than the original balloon altitude by; (a) an amount sufficient to encompass any probable final balloon altitude, or (b) by at least 200 ft if it is expected that the final balloon altitude will be lower than the original value.

No prints of cable parameters at intermediate altitudes between the balloon and surface are provided for two reasons. Simple print commands would produce excessively printed tape since many interim solutions from balloon to surface can be made before the correct solution is found. Logic allowing printing of only the final solution's intermediate altitude cable parameters requires excessive program steps. The balloon altitude and cable length (or cable hor. or tension 0 if no solution to the surface is possible) are printed for each altitude being tried as a monitoring aid. When the correct altitude is found, the balloon and surface (winch) conditions are printed. If the cable space position, shape, and other parameters are needed, the resulting balloon altitude and other input data can be used in Program No. 77.007 for print only or in Program No. 77.007P for print and/or plot.

Since incorrect high balloon altitudes are tried in certain cases in the search before the correct altitude is found, the chances of finding either zero cable tension or horizontal cable conditions above the ground are much greater in operating this program. To prevent this condition from improperly stopping the program, special treatment had to be evolved. When one of these conditions is reached for the first or second (higher) balloon altitude run, the balloon altitude is dropped by 10 percent of its original height above ground and the first and higher second balloon altitude calculations are made again from this point. If either of these runs yield the cable tension zero or horizontal above ground message, another 10 percent drop is made. In this way the proper solution is approached by either at least three rising balloon altitude increments or any number of decreasing balloon altitudes. If no final solution occurs in this procedure before the tension zero or cable horizontal condition is signaled on a third or higher altitude loop, it can be properly concluded that no solution exists and such final message is given on the tape. Also if the large 10 percent decrements in balloon height finally place the balloon at the surface, the no-solution message is given.

3.4.2 RERUN OPTIONS

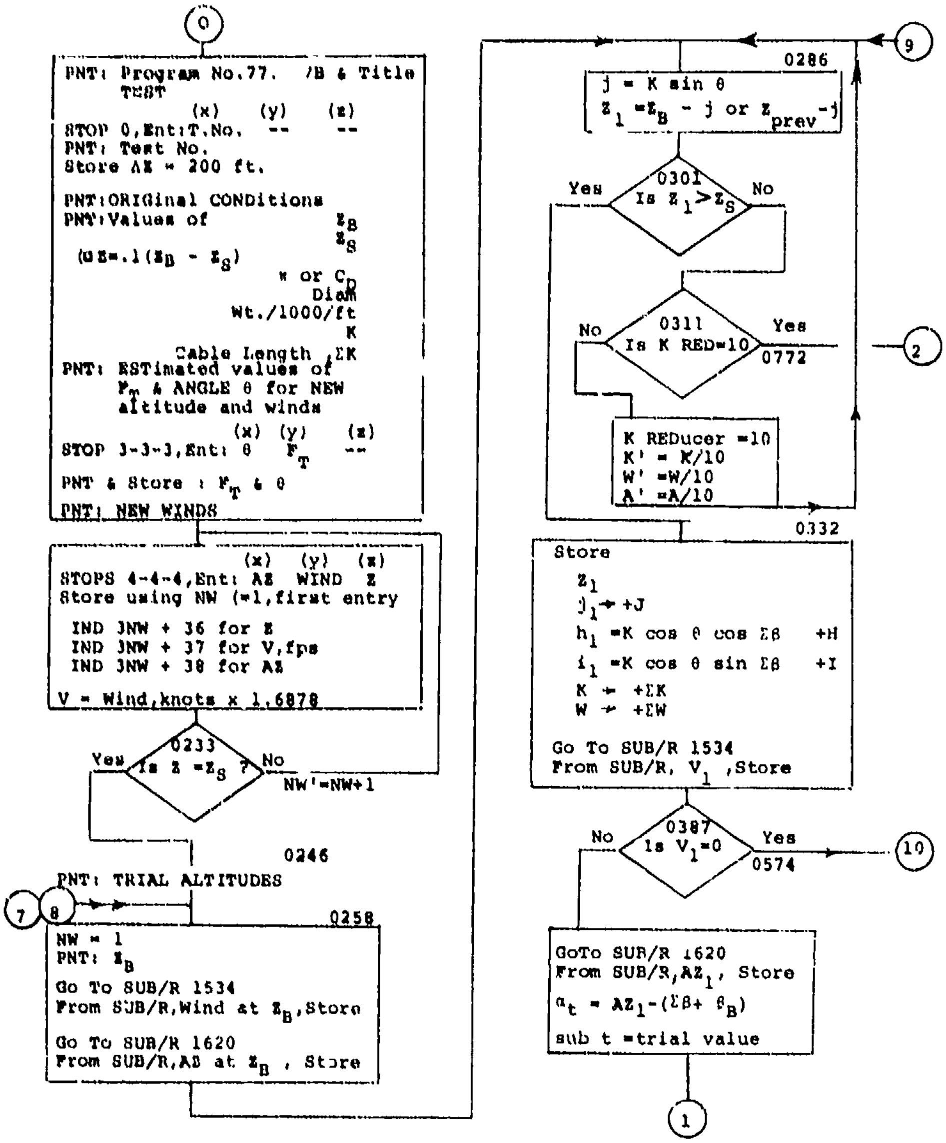
At the conclusion of the printout of the final solution of balloon altitude, wind magnitude and direction and all of the same surface parameters that are obtained with Programs 77.007 and 77.007P, a choice of two rerun options is provided. The option numbers 0 and 3 have the same nature of meaning as those in the basic program, 77.007.

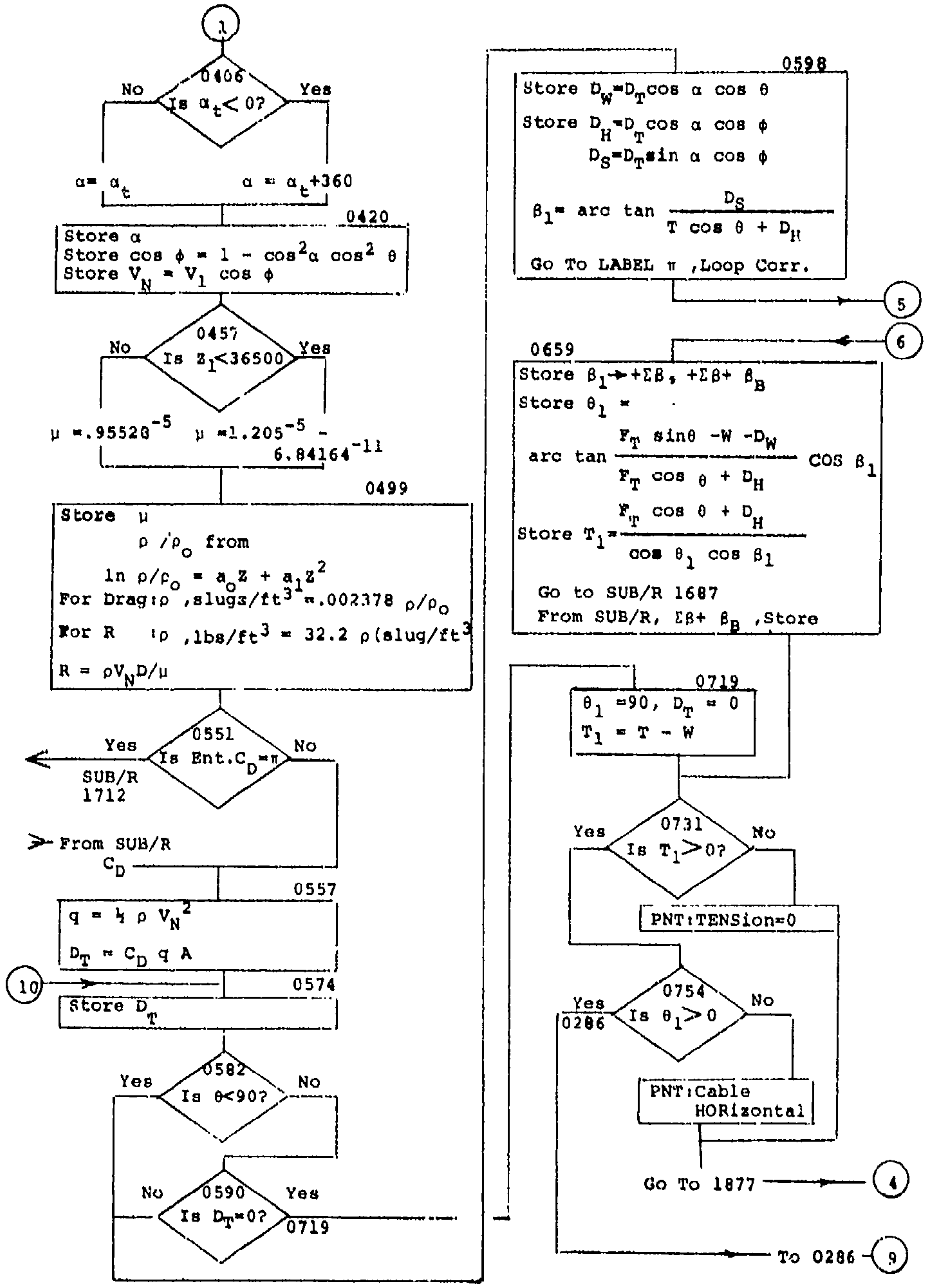
77.007B

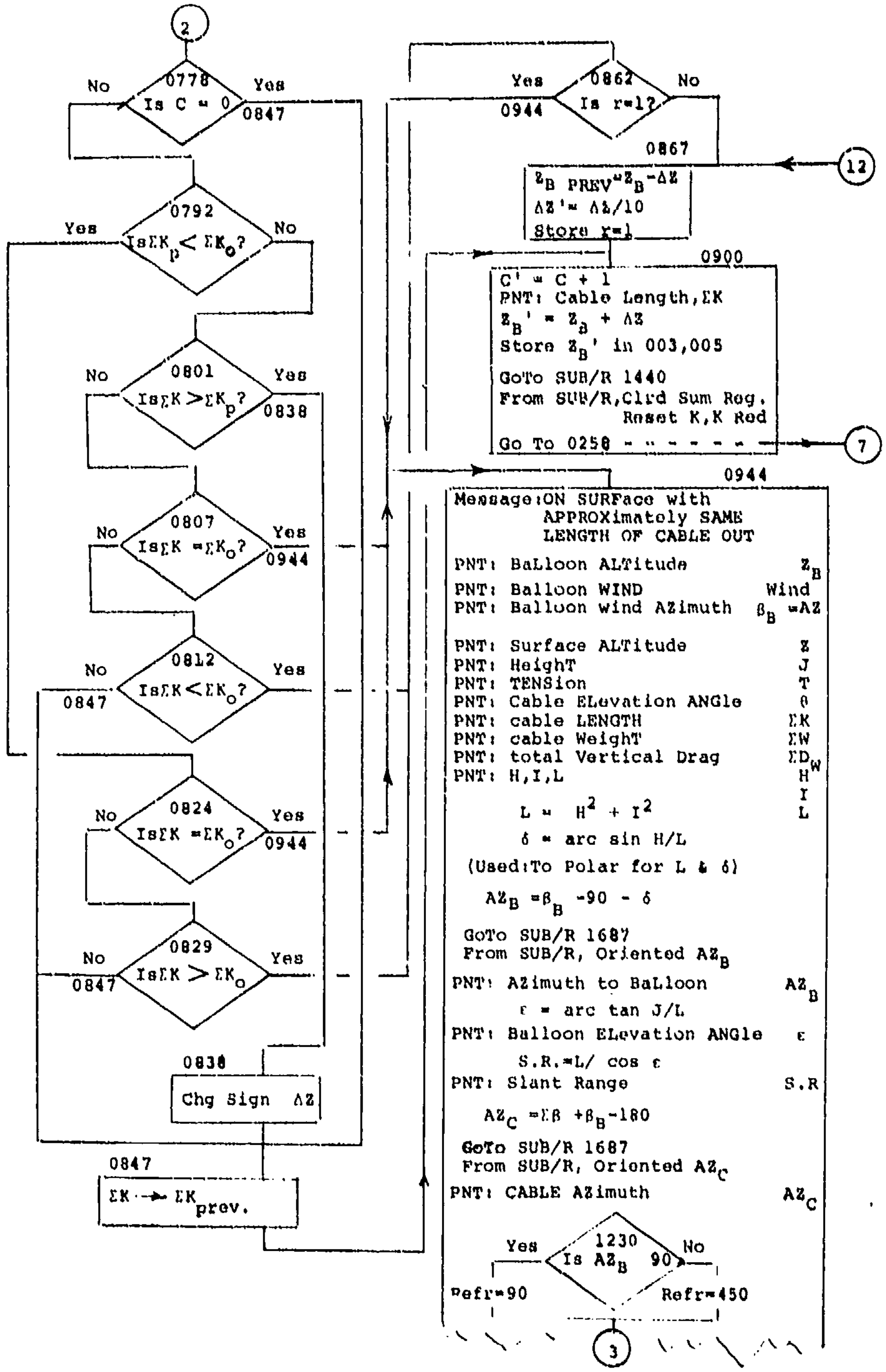
Option 0 calls for a completely new problem which must be solved in 77.007. Hence, if this option is chosen the program cards for 77.007 must be available for the automatic read-in built into this program.

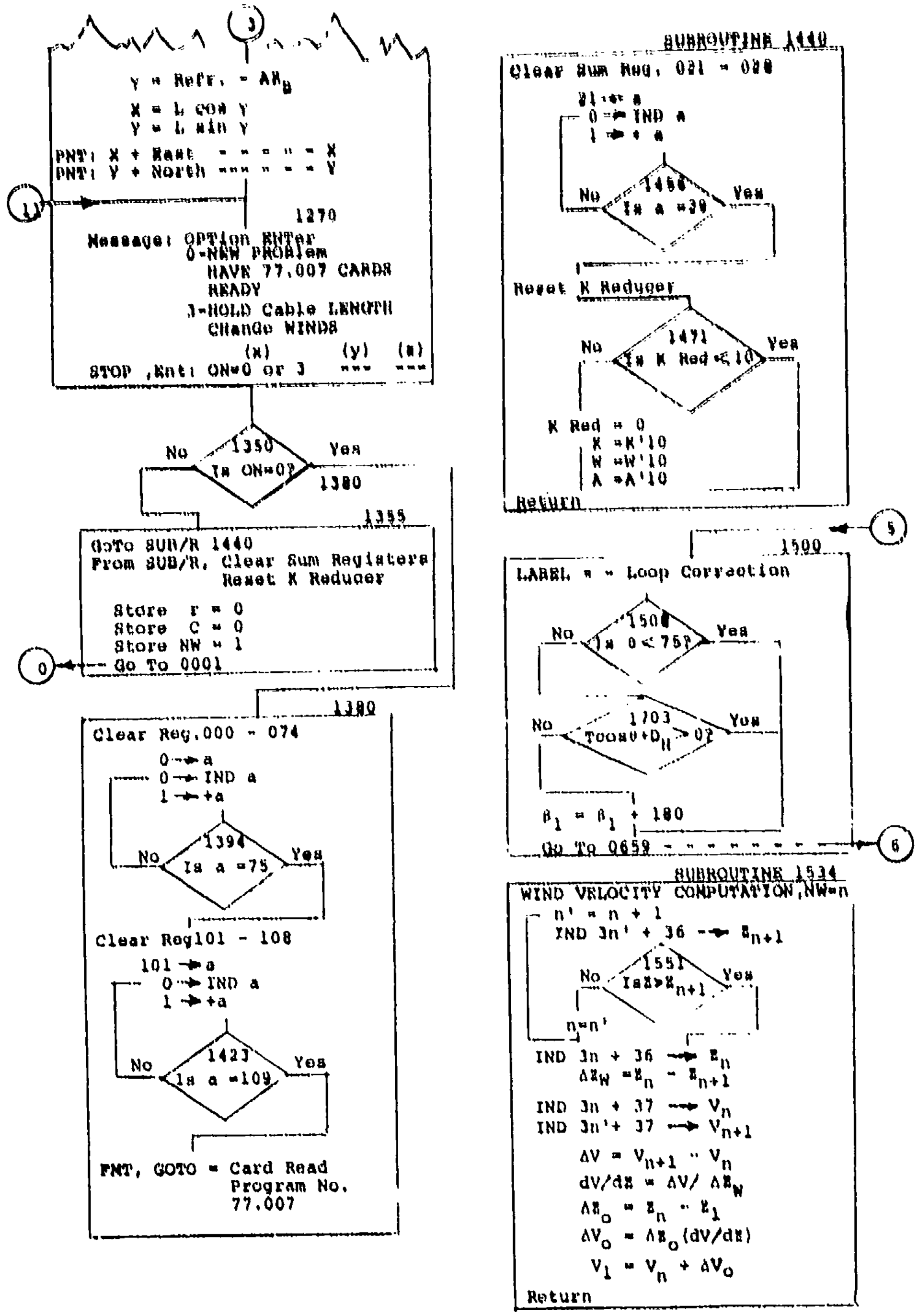
Option 3 calls for the cable length still to be held fixed and new winds to be introduced. Use of this option then causes a return to the start of 77.007B for re-entry of F_T, θ, and winds.

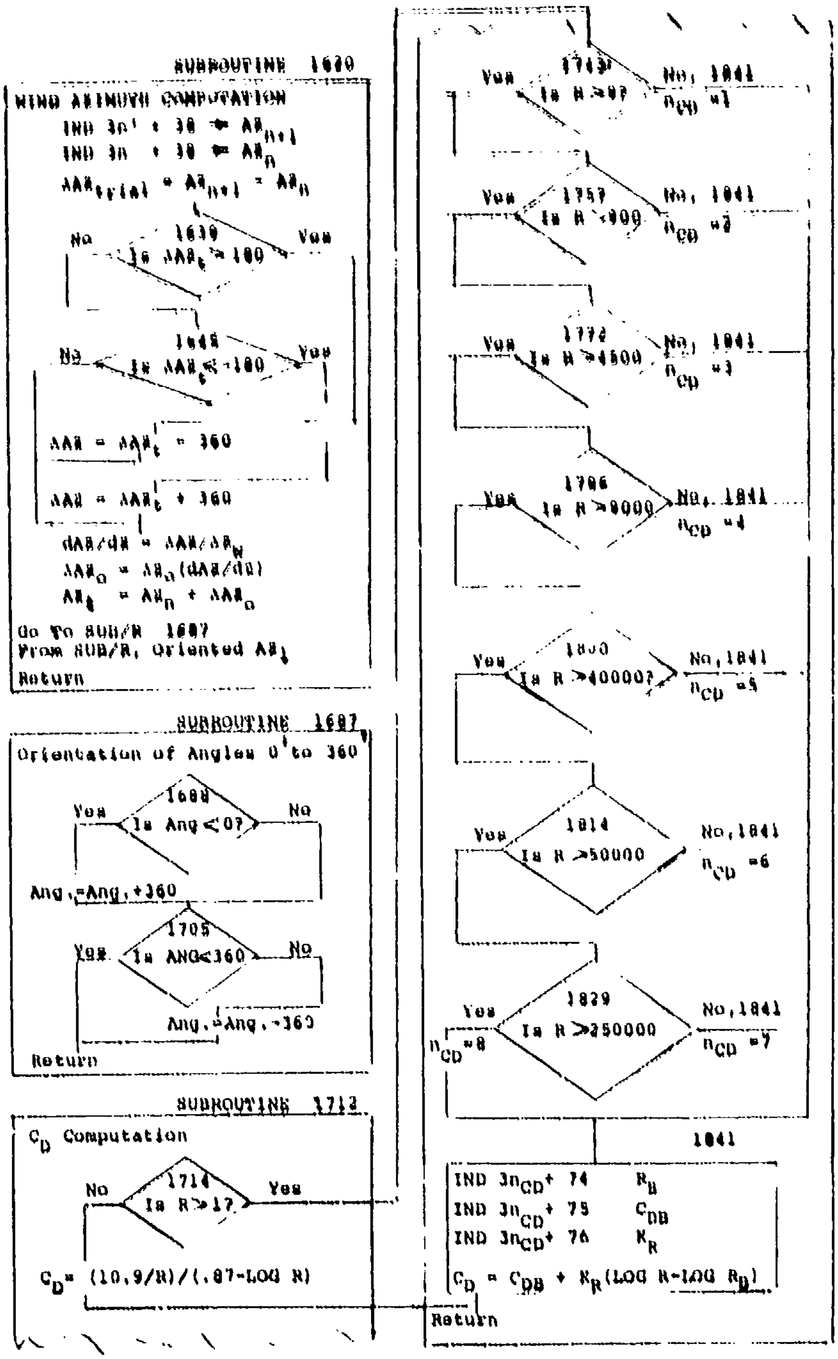
3.4.3 FLOW CHART

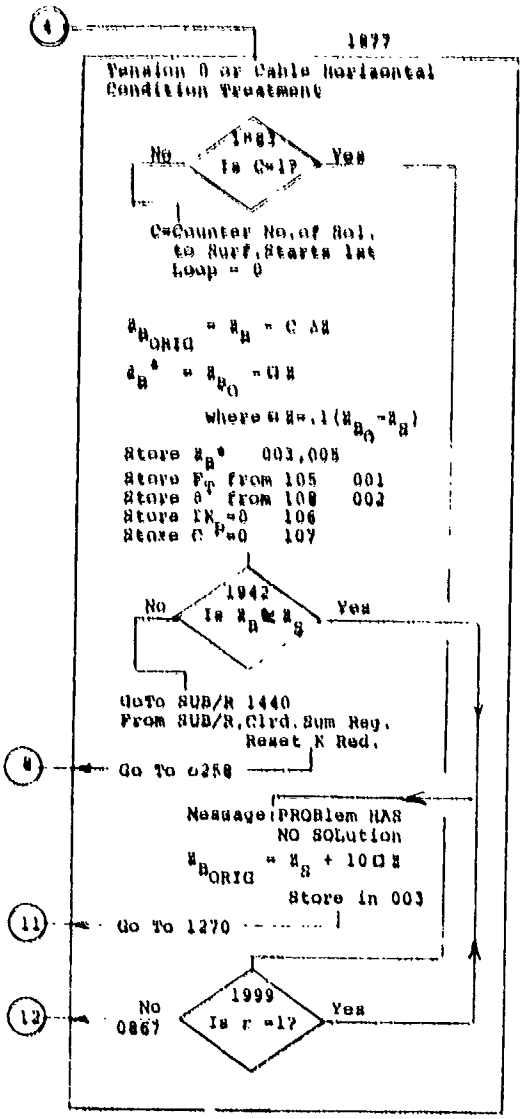



1
0406
Is α_t < 0?
No
Yes
α = α_t
α = α_t+360
0420
Store α
Store cos φ = 1 - cos²α cos² θ
Store V_N = V_1 cos φ
0457
Is Z_1<36500
No
Yes
μ =.95528^-5
μ =1.205^-5 - 6.84164^-11
0499
Store μ
ρ /ρ_o from
ln ρ/ρ_o = a_oZ + a_1Z²
For Drag:ρ ,slugs/ft³ =.002378 ρ/ρ_o
For R :ρ ,lbs/ft³ = 32.2 ρ(slug/ft³
R = ρV_ND/μ
0551
Is Ent.C_D=π
Yes
No
SUB/R 1712
From SUB/R C_D
0557
q = ½ ρ V_N²
D_T = C_D q A
10
0574
Store D_T
0582
Is θ<90?
Yes
No
0590
Is D_T=0?
No
Yes
0719
0598
Store D_W=D_Tcos α cos θ
Store D_H=D_Tcos α cos φ
D_S=D_Tsin α cos φ
β_1= arc tan D_S / (T cos θ + D_H)
Go To LABEL π ,Loop Corr.
5
6
0659
Store β_1→+Σβ, +Σβ+ β_B
Store θ_1 =
arc tan (F_T sinθ -W -D_W) / (F_T cos θ + D_H) cos β_1
Store T_1= (F_T cos θ + D_H) / (cos θ_1 cos β_1)
Go to SUB/R 1687
From SUB/R, Σβ+ β_B ,Store
0719
θ_1 =90, D_T = 0
T_1 = T - W
0731
Is T_1>0?
Yes
No
PNT:TENSion=0
0754
Is θ_1>0
Yes
0286
No
PNT:Cable HORizontal
Go To 1877
4
To 0286
9


2
0778
Is C = 0
No
Yes
0847
0792
IsΣK_p < ΣK_o?
Yes
No
0801
IsΣK > ΣK_p?
No
Yes
0838
0807
IsΣK = ΣK_o?
No
Yes
0944
0812
IsΣK < ΣK_o?
No
Yes
0847
0824
IsΣK = ΣK_o?
No
Yes
0944
0829
IsΣK > ΣK_o
No
Yes
0847
0838
Chg Sign ΔZ
0847
ΣK → ΣK prev.
0862
Is r=1?
Yes
No
0944
0867
Z_B PREV = Z_B - ΔZ
ΔZ' = ΔZ/10
Store r=1
12
0900
C' = C + 1
PNT: Cable Length, ΣK
Z_B' = Z_B + ΔZ
Store Z_B' in 003,005
GoTo SUB/R 1440
From SUB/R, Clrd Sum Reg. Reset K,K Red
Go To 0258
7
0944
Message: ON SURFace with APPROXimately SAME LENGTH OF CABLE OUT
PNT: BaLloon ALTitude Z_B
PNT: Balloon WIND Wind
PNT: Balloon wind AZimuth β_B = AZ
PNT: Surface ALTitude Z
PNT: HeighT J
PNT: TENSion T
PNT: Cable ELevation ANGle θ
PNT: cable LENGTH ΣK
PNT: cable WeighT ΣW
PNT: total Vertical Drag ΣD_W
PNT: H,I,L H I L
L = H^2 + I^2
δ = arc sin H/L
(Used: To Polar for L & δ)
AZ_B = β_B - 90 - δ
GoTo SUB/R 1687
From SUB/R, Oriented AZ_B
PNT: AZimuth to BaLloon AZ_B
ε = arc tan J/L
PNT: Balloon ELevation ANGle ε
S.R. = L/ cos ε
PNT: Slant Range S.R
AZ_C = Σβ + β_B - 180
GoTo SUB/R 1687
From SUB/R, Oriented AZ_C
PNT: CABLE AZimuth AZ_C
1230
Is AZ_B 90
Yes
No
Refr=90
Refr=450
3


3
Y = Refr. - AK
X = L cos Y
Y = L sin Y
PNT: X + East
PNT: Y + North
1
1270
Message: OPTion ENTer
0-NEW PROBlem
HAVE 77.007 CARDS
READY
3-HOLD Cable LENGTH
CHange WINDS
(x) (y) (z)
STOP ,Ent: ON=0 or 3
No
1350
Is ON=0?
Yes
1380
1355
GoTo SUB/R 1440
From SUB/R, Clear Sum Registers
Reset K Reducer
Store r = 0
Store C = 0
Store NW = 1
Go To 0001
0
1380
Clear Reg.000 - 074
0 → a
0 → IND a
1 → +a
No
1394
Is a =75
Yes
Clear Reg101 - 108
101 → a
0 → IND a
1 → +a
No
1423
Is a =109
Yes
PNT, GOTO = Card Read
Program No.
77.007
SUBROUTINE 1440
Clear Sum Reg. 021 - 028
21 → a
0 → IND a
1 → + a
No
1456
Is a =29
Yes
Reset K Reducer
No
1471
Is K Red ≤ 10
Yes
K Red = 0
K = K'10
W = W'10
A = A'10
Return
5
1500
LABEL = - Loop Correction
No
1500
Is 0 < 75?
Yes
No
1703
Tcosθ+D > 0?
Yes
β1 = β1 + 180
Go To 0659
6
SUBROUTINE 1534
WIND VELOCITY COMPUTATION,NW=n
n' = n + 1
IND 3n' + 36 → Z n+1
No
1551
Is Z > Z n+1
Yes
n=n'
IND 3n + 36 → Zn
ΔZW = Zn - Zn+1
IND 3n + 37 → Vn
IND 3n'+ 37 → Vn+1
ΔV = Vn+1 - Vn
dV/dZ = ΔV/ ΔZW
ΔZo = Zn - Z1
ΔVo = ΔZo(dV/dZ)
V1 = Vn + ΔVo
Return


SUBROUTINE 1620
WIND AZIMUTH COMPUTATION
Do To SUB/R 1687
From SUB/R, Oriented AZ
Return
SUBROUTINE 1687
Orientation of Angles 0 to 360
Yes
No
Is Ang < 0?
Ang.=Ang.+360
1705
Is ANG<360
Ang.=Ang.-360
Return
SUBROUTINE 1712
C_D Computation
No
1714
Is R>17
Yes
C_D= (10.9/R)/(.87-LOG R)
Return
Is R >400000?
No,1841
1814
Is R >50000
No,1841
1829
Is R >250000
No,1841
1841
IND 3n_CD+ 74 R_B
IND 3n_CD+ 75 C_DB
IND 3n_CD+ 76 K_R
C_D = C_DB + K_R(LOG R-LOG R_B)
Return



4
1977
Tension 0 or Cable Horizontal
Condition Treatment
1983
Is C=1?
No
Yes
C=Counter No.of Sol.
to Surf.Starts 1st
Loop = 0
Store
003,008
Store
from 105
001
Store
from 108
002
Store
106
Store
107
1942
No
Yes
GoTo SUB/R 1440
From SUB/R,Clrd.Sum Reg.
Reset K Red.
8
Go To 0258
Message:PROBlem HAS
NO SOLution
Store in 003
11
Go To 1270
1999
12
No
0867
Yes

3.4.4 OPERATING INSTRUCTIONS

Four sides of the two Program No. 77.007B cards will have been loaded after OPT ENT 3 was specified according to Program No. 77.007 instructions—Section 3.3.4.

KEY STROKES	ENTRIES (X)	ENTRIES (Y)	ENTRIES (Z)	PRINTING
$\boxed{\text{END}}$				
$\boxed{\text{FIX}}$ $\boxed{2}$, $\boxed{3}$, --- (No. of decimal places desired)				
$\boxed{\text{CONT}}$				PROG#77.007B CONST. CABLE LENGTH TEST #
STOP 0, Enter:	Test No.	---	---	
$\boxed{\text{CONT}}$				T. No. ORIG. COND Z_B Z_S π or C_D D Wt/1000 K (Cable Length) ΣK_o EST. NEW FT/ANGLE
STOP 3, Enter:	θ (Angle of F_T to Horizon, deg)	F_T (Balloon Tot. Force to Horizon, lbs)	--	
$\boxed{\text{CONT}}$				F_T θ NEW WINDS
STOP 4, Enter:	AZ (Azimuth of Wind, deg)	WIND (Wind, knots)	Z (Altitude, ft MSL)	
$\boxed{\text{CONT}}$				Z WIND AZ

See Sections 3.4.1 and 3.4.5 for notes on first or highest altitude entry. Stop 4's will repeat until last set of entries of Z_S are inserted. A total of 12 sets of entries may be made. After Z_S entry and CONT:

TRIAL ALT

Z_B
ΣK

If no cable-horizontal or tension-zero conditions are encountered, the progression of trial altitudes and their respective computed cable lengths are printed as shown:

Z'
$\Sigma K'$

Z''
$\Sigma K''$

—

—

When a condition is met where the computed cable length equals the original length, the following printout is made including the abbreviated names and values of the parameters shown.

ON SURF. W. APPROX
SAME LENGTH OF
CABLE OUT

BLN. ALT	Z_B	Balloon Altitude, ft MSL
B. WIND	Wind	Balloon Wind, knots
B. AZ	β_B=AZ	Balloon Wind Azimuth, Deg. (Also Balloon Pointing Azimuth)
S. ALT	Z	Ending Surface Altitude, ft MSL
HT	J	Vertical Height, ft
TENS	T	Cable Tension (Winch), lbs
C. EL. ANG	θ	Cable Elevation Angle (Winch), lbs
LENGTH	ΣK	Cable Length, ft
WT	ΣW	Cable Weight, lbs
V. D	ΣD_W	Total Vertical Drag Comp., lbs
H, I, L	H	Total H. Distances on Y_B or Y_W axis, ft
	I	Total H. Distance on X_B or X_W axis, ft
	L	Min. Direct H. Dist. to balloon
AZ TO BLN	AZ_B	Azimuth Angle to Balloon, deg
B. EL. ANG	ϵ	Elevation Angle to Balloon, deg
S. R	SR	Slant Range to Balloon, ft
CABLE AZ	AZ_C	Azimuth Angle of Cable (out of winch), deg
X + E	X	X Coordinate to Balloon, ft
Y + N	Y	Y Coordinate to Balloon, ft

If cable-horizontal or tension-zero conditions are encountered, one of the two types of printouts shown will occur in the succession of trial altitudes before the final solution is found and printed.

Z_B C. HOR	Z_B TENS 0
Z* C. HOR	Z* TENS 0
Z*' ΣK'	Z*' Tens 0
Z*'' ΣK''	Z*'' ΣK''
Z*''' ΣK'''	Z*''' ΣK'''

At this point the problem is solved. Two optional ways of continuing are now provided by the following Message and STOP.

OPT END

0-NEW PROB-HAVE 77.007 CARDS READY

3-HOLD C. LENGTH CHG. WINDS

STOP, Enter: 0 or 3 in (X)

CONT

If 0 is Entered - New Problem—Use this for a completely new problem requiring use of Program No. 77.007. Insert side 1 of 77.007 Program card before pressing the CONT. Continue feeding additional 3 sides of cards and follow instructions in Section 3.3.4. The permanent storage registers containing density and drag coefficient constants will be retained so that no read-in of the data cards will be required—same retention in either direction between the two programs.

If 3 is Entered - Hold Cable Length, Change Winds—In this case, the original cable length will be retained, the summary registers will be cleared, and a return to the start of 77.007B will be made. The first trial balloon altitude will then be the final balloon altitude solved in the previous problem.

Note No. 1: In order to speed up the process where the final balloon altitude will be significantly different from the starting balloon altitude, an altitude "guess" value may be inserted into Register 003 at the Test Number STOP for an original problem run or at the OPT ENT STOP for other runs.

Note No. 2: If incorrect data is believed to have been entered do not press STOP END to restart program. For correct and safe clearing of registers, press the following:

77.007B

STOP

GO TO

1

2

7

0

After OPT ENT message is printed, enter 3 , then:

CONT

3.4.5 INPUT DATA FORM

Test No. __________ Date: __________________ Notes: __________________________

ORIGINAL VALUES						
1. Balloon Altitude	=	ft MSL	4.	F_T	=	lbs
2. Wind at Balloon	=	knots	5.	θ	=	deg
3. Azimuth of W. at Balloon	=	degree	6.	Cable Length	=	ft

ESTIMATES		#1	#2	#3	#4	#5
7. New Balloon Altitude	Z_B					
8. New Wind at Balloon	Wind					
9. New AZ of W at Balloon	AZ					

INPUT							
STOP 3	Total Force, F_T	lbs					
	Angle of F_T, θ	deg					

STOP 4

NEW WIND PROFILE

1. Z Wind AZ	5. Z Wind AZ	9. Z Wind AZ
2. Z Wind AZ	6. Z Wind AZ	10. Z Wind AZ
3. Z Wind AZ	7. Z Wind AZ	11. Z Wind AZ
4. Z Wind AZ	8. Z Wind AZ	12. Z Wind AZ

Up to 12 wind points may be specified. First point must be a minimum of 200 ft above original balloon altitude (1) or above best estimate of higher balloon altitude (7), whichever is higher. Last point must be for surface altitude.

3.4.6 PROGRAM NO. 77.007B LISTING

STEP	KEY	CODE	X	Y	Z	STEP	KEY	CODE	X	Y	Z
000 0	CNT					0	0				
1	FMT					1	3				
2	FMT					2	FMT				
3	P					3	FMT				
4	R					4	0				
5	O					5	R				
6	G					6	I				
7	#					7	G				
8	7					8	•				
9	7					9	C				
0	•					0	O				
1	0					1	N				
2	0					2	D				
3	7					3	FMT				
4	B					4	X( )				
5	CLR					5	3		Z_B		
6	C					6	PNT				
7	O					7	X→				
8	N					8	5				
9	S					9	↑			Z_B	
0	T					0	X( )				
1	•					1	4		Z_S	Z_B	
2	C					2	PNT				
3	A					3	−			Z_B-Z_S	
4	B					4	1				
5	L					5	0		10	Z_B-Z_S	
6	E					6	÷			ΔZ	
7	CLR					7	y→				
8	L					8	1				
9	E					9	0				
0	N					0	1				
1	G					1	X( )				
2	T					2	1				
3	H					3	2		TarG		
4	CLR					4	PNT				
5	T					5	X( )				
6	E					6	1				
7	S					7	1		D, ft		
8	T					8	↑			D	
9	CNT					9	1				
0	#					0	2		12	D	
1	FMT					1	×			D in	
2	STOP		T. No			2	↓		D		
3	PNT					3	PNT				
4	0					4	X( )				
5	2					5	9		W		
6	0					6	↑			W	
7	0		Az=200			7	X( )				
8	X→					8	6		K	W	
9	/					9	÷			W/ft	

STEP	KEY	CODE	X	Y	Z	STEP	KEY	CODE	X	Y	Z
010 0	1					0	X→				
1	0					1	1				
2	0					2	0				
3	0		1000	Wt/ft		3	8				
4	X			Wt		4	FMT				
5	↓		Wt			5	FMT				
6	PNT					6	N				
7	X( )					7	E				
8	6		K			8	W				
9	PNT					9	CNT				
0	X( )					0	W				
1	1					1	I				
2	0					2	N				
3	2		ΣK_0			3	D				
4	PNT					4	S				
5	FMT					5	FMT				
6	FMT					016 6	4		4		
7	E					7	↑		4	4	
8	S					8	↑		4	4	4
9	T					9	STOP		AZ	WIND	Z
0	•					0	R↑		Z	AZ	WIND
1	N					1	PNT				
2	E					2	R↑		WIND	Z	AZ
3	W					3	PNT				
4	CNT					4	R↑		AZ	WIND	Z
5	F					5	PNT				
6	T					6	PNT				
7	/					7	y→				
8	A					8	3				
9	N					9	5				
0	G					0	X→				
1	L					1	3				
2	E					2	6				
3	FMT					3	b		NW		Z
4	3		3			4	X⇄Y			NW	Z
5	↑		3	3		5	3		3		
6	↑		3	3	3	6	X			3NW	
7	STOP		θ	F_T		7	3				
8	X⇄Y		F_T	θ		8	6		36	3NW	
9	PNT					9	+			3NW+36	Z
0	X→					0	y→				
1	1					1	a				
2	X→					2	R↑		Z		
3	1					3	X→				
4	0					4	IND				
5	5					5	a				
6	X⇄Y		θ	F_T		6	X( )				
7	PNT					7	3				
8	X→					8	5		WIND		
9	2					9	X⇄Y			WIND	

STEP	KEY	CODE	X	Y	Z	STEP	KEY	CODE	X	Y	Z
020 0	1					0	1				
1	.					1	A				
2	6					2	L				
3	8					3	CNT				
4	7					4	A				
5	8		1.68	WIND		5	L				
6	X			V, fps		6	T				
7	1		1			7	FMT				
8	X→					025 8	1				
9	+					9	X→				
0	a					0	b				
1	y→					1	X( )				
2	IND					2	5		Z_B		
3	a					3	PNT				
4	X→		1			4	GoTo				
5	+					5	SUB/R				
6	a					6	1				
7	X( )					7	5				
8	3					8	3				
9	6		AZ			9	4				
0	X→					0	y→			V_B	
1	IND					1	1				
2	a					2	0				
3	2		2			3	4				
4	X→					4	GoTo				
5	−					5	SUB/R				
6	a					6	1				
7	X( )					7	6				
8	IND					8	2				
9	a		Z			9	0				
0	↑			Z		0	y→			$AZ_B = \beta_B$	
1	X( )					1	2				
2	4		Z_B	Z		2	7				
3	X=Y					3	y→				
4	0					4	3				
5	2					5	0				
6	4					028 6	X( )				
7	6					7	2		θ		
8	1		1			8	SIN X		SIN θ		
9	X→					9	↑			SIN θ	
0	+					0	X( )				
1	b					1	6		K	SIN θ	
2	GoTo					2	X			J	
3	1					3	X( )				
4	0					4	5		Z	J	
5	6					5	X⇄Y		J	Z	
024 6	FMT					6	−		J	Z'	
7	FMT					7	↑			J	Z'
8	7					8	X( )				
9	R					9	4		Z_S	J	Z'

STEP	KEY	CODE	X	Y	Z
030 0	R↑		Z'	Zs	J
1	X>Y				
2	0				
3	3				
4	3				
5	2				
6	X()				
7	7		K RED	Zs	J
8	X⇄Y		Zs	K RED	
9	1				
0	0		10		
1	X=Y				
2	0				
3	7				
4	7				
5	2				
6	X→				
7	7				
8	X→				
9	÷				
0	6				
1	X→				
2	÷				
3	9				
4	X→				
5	÷				
6	1				
7	0				
8	GoTo				
9	2				
0	8				
1	6				
033 2	X→		Z'	Zs	J
3	5				
4	R↑		J	Z'	Zs
5	X→				
6	+				
7	2				
8	3				
9	X()				
0	2		θ		
1	cos X		cos θ		
2	↑			cos θ	
3	X()				
4	6		K	cos θ	
5	X→				
6	+				
7	2				
8	4				
9	X			Kcosθ	

STEP	KEY	CODE	X	Y	Z
0	X()				
1	2				
2	8		∠β	Kcosθ	
3	cos X		cos∠β	Kcosθ	
4	X⇄Y		Kcosθ	cos∠β	
5	X		Kcosθ	h	
6	Y→				
7	+				
8	2				
9	1				
0	↑			Kcosθ	
1	X()				
2	2				
3	8		∠β	Kcosθ	
4	SIN X		SIN∠β	Kcosθ	
5	X			l	
6	Y→				
7	+				
8	2				
9	2				
0	X()				
1	9		W		
2	X→				
3	+				
4	2				
5	5				
6	GoTo				
7	SUB/R				
8	1				
9	5				
0	3				
1	4				
2	Y→			Y fps	
3	0				
4	1				
5	3				
6	0		0	Y	
7	X=Y				
8	0				
9	5				
0	7				
1	4				
2	GoTo				
3	SUB R				
4	1				
5	6				
6	2				
7	0				
8	Y→			Az	
9	1				

STEP	KEY	CODE	X	Y	Z
040 0	4			AZ	
1	X( )				
2	2				
3	7		ΣB+/Ga	AZ	
4	−			α_t	
5	0		0	α_t	
6	X>Y				
7	0				
8	4				
9	1				
0	6				
1	GoTo				
2	0				
3	4				
4	2				
5	0				
041 6	3				
7	6				
8	0		360	α_t	
9	+			α	
042 0	y→				
1	1				
2	5				
3	↓		α		
4	cos X		$\cos\alpha$		
5	X^2		$\cos^2\alpha$		
6	↑			$\cos^2\alpha$	
7	X( )				
8	2		θ		
9	cos X		$\cos\theta$		
0	X^2		$\cos^2\theta$	$\cos^2\alpha$	
1	X			$\cos^2\cos^2$	
2	1		1		
3	X⇌Y		$\cos^2\cos^2$	1	
4	−			$\cos^2\phi$	
5	↓		$\cos^2\phi$		
6	$\sqrt{X}$		$\cos\phi$		
7	X→				
8	2				
9	9				
0	↑			$\cos\phi$	
1	X( )				
2	1				
3	3		Υ	$\cos\phi$	
4	X			V_N	
5	y→				
6	1				
7	6				
8	X( )				
9	5		Z		

STEP	KEY	CODE	X	Y	Z
0	↑		Z	Z	
1	↑		Z	Z	Z
2	3				
3	6				
4	5				
5	0				
6	0		36500	Z	Z
7	X>Y				
8	0				
9	4				
0	7				
1	6				
2	•				
3	9				
4	6				
5	5				
6	2				
7	8				
8	ENT E				
9	5				
0	CHG S		μ	Z	Z
1	GoTo				
2	0				
3	4				
4	9				
5	9				
047 6	6				
7	•				
8	8				
9	4				
0	1				
1	6				
2	4				
3	ENT E				
4	1				
5	1				
6	CHG S		6.8^{-11}	Z	Z
7	X			6.8^{-11} Z	Z
8	1				
9	•				
0	2				
1	0				
2	5				
3	ENT E				
4	5				
5	CHG S		1.2^{-5}		
6	X⇌Y		6.8^{-11}Z	1.2^{-5}	Z
7	−			μ	Z
8	↓		μ	Z	Z
049	X→				

050	0	3		μ	Z	Z		0	π		π	π or C_D	R
	1	3						1	X=Y				
	2	X( )						2	SUB/R				
	3	7						3	1				
	4	5		a_1	Z			4	7				
	5	X			a_1Z	Z		5	1				
	6	X( )						6	2				
	7	7						7	X( )			C_D	
	8	6		a_0	a_1Z	Z		8	1				
	9	+			a_0+a_1Z			9	6		V_N		
	0	↓		a_0+a_1Z	Z	Z		0	X^2		V_N^2	C_D	
	1	X			ln ρ/ρ0			1	↑			V_N^2	C_D
	2	↓		ln ρ/ρ0				2	X( )				
	3	e^X		ρ/ρ0				3	3				
	4	↑			ρ/ρ0			4	2		ρ	V_N^2	
	5	•						5	X			ρV_N^2	
	6	0						6	2		2		
	7	0						7	÷			q	C_D
	8	2						8	↓		q	C_D	
	9	3						9	X			qC_D	
	0	7						0	X( )				
	1	8		ρ_0	ρ/ρ0			1	1				
	2	X			ρ slug/ft^3			2	0		A	qC_D	
	3	y→						3	X			D_T	
	4	0					057	4	y→				
	5	3						5	3				
	6	2						6	1				
	7	3						7	X( )				
	8	2						8	2		θ	D_T	
	9	•						9	↑			θ	D_T
	0	1						0	9				
	1	7						1	0		90	θ	D_T
	2	4		32.2	ρ			2	X>Y				
	3	X			ρ lb/ft^3			3	0				
	4	X( )						4	5				
	5	1						5	9				
	6	6		V_N	ρ			6	8				
	7	X			ρV_N			7	0		0	θ	D_T
	8	X( )						8	R↑		D_T	0	θ
	9	1						9	X⇄Y		0	D_T	θ
	0	1		D	$\rho V_N D$			0	X=Y				
	1	X			ρV_N			1	0				
	2	X( )						2	7				
	3	3						3	1				
	4	3		μ	$\rho V_N D$			4	9				
	5	÷			R			5	↓		D_T	θ	
	6	X( )						6	X⇄Y		θ	D_T	
	7	1						7	↑			θ	D_T
	8	2		π or C_D	R		059	8	↓		θ	D_T	
	9	↑			π or C_D	R		9	COS X		cos θ	D_T	

Step	Key	Code			
060	X			$D_T\cos$ [illegible]	
1	X( )				
2	/				
3	5		α		
4	COS X		cos α	$D_T\cos$ [illegible]	
5	X			D_W	
6	X⇄Y		D_W	cos α	
7	X→				
8	/				
9	7				
0	X→				
1	+				
2	2				
3	6				
4	X( )				
5	2				
6	9		cos φ	cos α	
7	X			$D_W\cos\alpha$	
8	↑			cos φ	cos α
9	X( )				
0	2				
1	/		D_T	cos φ	cos α
2	R↑		cos α	D_T	cos φ
3	X			D_N	
4	y→				
5	/				
6	8				
7	X( )				
8	3				
9	/		D_T	D_H	cos φ
0	R↑		cos φ	D_T	
1	X			$D_T\cos\phi$	
2	X( )				
3	/				
4	5		α		
5	SIN X		SIN α	$D_T\cos\phi$	
6	X			D_S	
7	X( )				
8	2		θ		
9	cos X		cos θ	D_S	
0	↑			cos θ	D_S
1	X( )				
2	/		FcosT	cos θ	D_S
3	X			T cos θ	D_S
4	X( )				
5	/				
6	8		D_H		
7	+			T cos + D_H	D_S
8	↓		T cos + D_H	D_S	D_S
9	X→				

Step	Key	Code			
0	3				
1	0				
2	÷			[illegible] A_1	D_1
3	↓		tan A_1		
4	arc				
5	tan X		A_1		
6	GTO B				
7	LABEL				
8	π				
9	X→		A_1		
0	+				
1	2				
2	7				
3	X→				
4	+				
5	2				
6	8				
7	cos X		cos A_1		
8	↑			cos A_1	
9	X( )				
0	2		θ		
1	SIN X		SIN θ	cos A_1	
2	↑			SIN θ	cos A_1
3	X( )				
4	/		FcosT	SIN θ	
5	X			T SIN θ	
6	X( )				
7	9		W	T SIN θ	
8	−			T SIN − W	
9	X( )				
0	/				
1	7		D_W		
2	−			T SIN − W − D_W cos A_1	
3	X( )				
4	2				
5	0		T cos + D_H		
6	÷			( )/( )	cos A_1
7	↓		( )/( )	cos A_1	cos A_1
8	X			TAN θ_1	
9	↓		TAN θ_1	cos A_1	
0	arc				
1	TAN X		θ_1		
2	X→				
3	2				
4	cos X		cos θ_1	cos A_1	
5	X			cos θ_1 cos A_1	
6	X( )				
7	2				
8	0		T cos + D_H		
9	X⇄y		cos cos	T cos + D_H	

070	0	[illegible]			Y	
	1	[illegible]				
	2	/				
	3	Y( )			[illegible]	
	4	4				
	5	/				
	6	Go To				
	7	[illegible]				
	8	/				
	9	1				
	0	/				
	1	/				
	2	Y→			[illegible]	
	3	8				
	4	/				
	5	Go To				
	6	/				
	7	4				
	8	7				
071	9	X( )				
	0	/		T		
	1	↑			T	
	2	X( )				
	3	9		N	T	
	4	−			T_1	
	5	Y→				
	6	/				
072	7	X( )				
	8	/		T_1		
	9	↑			Y	
	0	0		0	T_1	
	1	X<Y				
	2	0				
	3	7				
	4	5				
	5	0				
	6	FMT				
	7	FMT				
	8	T				
	9	E				
	0	N				
	1	S				
	2	•				
	3	0				
	4	FMT				
	5	Go To				
	6	1				
	7	8				
	8	7				
	0	7				

[illegible]	0	X( )				
	1	2		Q_1		
	2	↑			Q_1	
	3	0		Q	Q_2	
	4	X≠Y				
	5	0				
	6	8				
	7	4				
	8	4				
	9	FMT				
	0	FMT				
	1	C				
	2	•				
	3	N				
	4	O				
	5	R				
	6	FMT				
	7	Go To				
	8	1				
	9	4				
	0	7				
	1	7				
077	2	X( )				
	3	1				
	4	0				
	5	7		C		
	6	↑			C	
	7	0		Q	C	
	8	X≠Y				
	9	0				
	0	8				
	1	4				
	2	7				
	3	X( )				
	4	1				
	5	0				
	6	2		ΣK_o		
	7	↑			ΣK_o	
	8	X( )				
	9	1				
	0	0				
	1	6		ΣK_p	ΣK_o	
	2	X<Y				
	3	0				
	4	8				
	5	2				
	6	1				
	7	↑			ΣK_p	ΣK_o
	8	X( )				
	0	2				

STEP	KEY	CODE	X	Y	Z
080 0	4		ΣK	ΣK_o	$\Sigma K'_o$
1	X>Y				
2	0				
3	8				
4	3				
5	8				
6	R↑		ΣK_o	$\Sigma K'$	ΣK_p
7	X=Y				
8	0				
9	9				
0	4				
1	4				
2	X>Y				
3	0				
4	8				
5	3				
6	8				
7	GoTo				
8	8				
9	4				
0	7				
082 1	X( )				
2	2				
3	4		ΣK	ΣK_o	
4	X=Y				
5	1				
6	9				
7	4				
8	4				
9	X>Y				
0	0				
1	8				
2	5				
3	8				
4	GoTo				
5	8				
6	4				
7	7				
083 8	X( )				
9	1				
0	0				
1	3		ΔZ		
2	CHGS		$-\Delta Z$		
3	X→				
4	1				
5	0				
6	3				
084 7	X( )				
8	2				
9	4		ΣK		

STEP	KEY	CODE	X	Y	Z
0	X→				
1	1				
2	0				
3	6		$\Sigma K \rightarrow \Sigma K_p$		
4	GoTo				
5	9				
6	0				
7	0				
085 8	X( )				
9	0		r		
0	↑			r	
1	1		1	r	
2	X=Y				
3	0				
4	9				
5	4				
6	4				
7	X( )				
8	3		Z_B		
9	↑			Z_B	
0	X( )				
1	1				
2	0				
3	3		ΔZ	Z_B	
4	−		ΔZ	Z_{BP}	
5	y→				
6	3				
7	↑			ΔZ	
8	1				
9	0		10	ΔZ	
0	÷			$\Delta Z'$	
1	1		1	$\Delta Z'$	
2	y→				
3	1				
4	0				
5	3				
6	X→		$1 = r$		
7	0				
8	CNT				
9	CNT				
0	CNT				
1	CNT				
2	CNT				
3	CNT				
4	CNT				
5	CNT				
6	CNT				
7	CNT				
8	CNT				
9	CNT				

STEP	KEY	CODE	x	y	z	STEP	KEY	CODE	x	y	z
090 0	X→		1			0	U				
1	+					1	R				
2	1					2	F				
3	0					3	•				
4	7					4	W				
5	X( )					5	•				
6	1					6	A				
7	0					7	P				
8	5		F_T			8	P				
9	X→					9	R				
0	1					0	O				
1	X( )					1	X				
2	1					2	S				
3	0					3	A				
4	8		θ			4	M				
5	X→					5	E				
6	2					6	CNT				
7	X( )					7	L				
8	2					8	E				
9	4		ΣK			9	N				
0	PNT					0	G				
1	PNT					1	T				
2	X( )					2	H				
3	3		Z_B			3	CNT				
4	↑			Z_B		4	O				
5	X( )					5	F				
6	1					6	CLR				
7	0					7	C				
8	3		ΔZ	Z_B		8	A				
9	+			Z_B'		9	B				
0	y→					0	L				
1	3					1	E				
2	y→					2	CNT				
3	5					3	O				
4	GoTo					4	U				
5	SUB/R					5	T				
6	1					6	CLR				
7	4					7	CLR				
8	4					8	B				
9	0					9	L				
0	GoTo					0	N				
1	2					1	•				
2	5					2	A				
3	8					3	L				
094 4	FMT					4	T				
5	FMT					5	FMT				
6	O					6	X( )				
7	N					7	3				
8	CNT					8	PNT		Z_B		
9	S					9	FMT				

STEP	KEY	CODE	X	Y	Z	STEP	KEY	CODE	X	Y	Z
100 0	FMT					0	2				
1	B					1	3		J		
2	•					2	PNT				
3	W					3	FMT				
4	I					4	FMT				
5	N					5	T				
6	D					6	E				
7	FMT					7	N				
8	X( )					8	S				
9	1					9	FMT				
0	0					0	X( )				
1	4		V			1	1		T		
2	↑			V		2	PNT				
3	1					3	FMT				
4	•					4	FMT				
5	6					5	C				
6	8					6	•				
7	7					7	E				
8	8		1.6878	V		8	L				
9	÷			WIND		9	•				
0	↓		WIND			0	A				
1	PNT					1	N				
2	FMT					2	G				
3	FMT					3	FMT				
4	B					4	X( )				
5	•					5	2		θ		
6	A					6	PNT				
7	Z					7	FMT				
8	FMT					8	FMT				
9	X( )					9	L				
0	3					0	E				
1	0		$B_B = AZ$			1	N				
2	PNT					2	G				
3	FMT					3	T				
4	FMT					4	H				
5	S					5	FMT				
6	•					6	X( )				
7	A					7	2				
8	L					8	4		ΣK		
9	T					9	PNT				
0	FMT					0	FMT				
1	X( )					1	FMT				
2	5		Z_s			2	W				
3	PNT					3	T				
4	FMT					4	FMT				
5	FMT					5	X( )				
6	H					6	2				
7	T					7	5		ΣW		
8	FMT					8	PNT				
9	X( )					9	FMT				

STEP	KEY	CODE	X	Y	Z	STEP	KEY	CODE	X	Y	Z
110 0	FMT					0	A				
1	V					1	Z				
2	•					2	•				
3	D					3	T				
4	FMT					4	O				
5	X( )					5	CNT				
6	2					6	B				
7	6					7	L				
8	PNT		ΣDW			8	N				
9	FMT					9	FMT				
0	FMT					0	PNT				
1	H					1	X→				
2	,					2	3				
3	I					3	6				
4	,					4	X( )				
5	L					5	2				
6	FMT					6	3		J		
7	X( )					7	↑			J	
8	2					8	X( )				
9	1					9	3				
0	PNT		H			0	5		L	J	
1	↑					1	÷		L	TAN E	
2	X( )			H		2	X⇄Y		TAN E	L	
3	2					3	ARC				
4	2		I	H		4	TANX		E	L	
5	PNT					5	FMT				
6	ToPoL		L	δ		6	FMT				
7	X→					7	B				
8	3					8	•				
9	5					9	E				
0	PNT					0	L				
1	X( )					1	•				
2	3					2	A				
3	0		β_B	δ		3	N				
4	↑			β_B	δ	4	G				
5	9					5	FMT				
6	0		90	β_B		6	PNT				
7	−			β_B-90	δ	7	COSX		COS E	L	
8	↓		β_B-90	δ		8	÷			S.R.	
9	X⇄Y		δ	β_B-90		9	FMT				
0	−			AZ_B		0	FMT				
1	GoTo					1	S				
2	SUB/R					2	•				
3	1					3	R				
4	6					4	FMT				
5	8					5	↓		S.R.		
6	7					6	PNT				
7	↓		AZ_B			7	X( )				
8	FMT					8	2				
9	FMT					9	7		$\Sigma B+\beta_B$		

STEP	KEY	CODE	X	Y	Z	STEP	KEY	CODE	X	Y	Z
120 0	↑			$\Sigma\beta+\beta_B$		0	R↑		SIN γ	L	X
1	1					1	X			Y	X
2	8					2	R↑		X		Y
3	0		180	$\Sigma\beta+\beta_B$		3	FMT				
4	—			AZ_C		4	FMT				
5	GoTo					5	X				
6	SUB/R					6	CNT				
7	1					7	+				
8	6					8	E				
9	8					9	FMT				
0	7					0	PNT				
1	↓		AZ_C			1	R↑		Y	X	
2	FMT					2	FMT				
3	FMT					3	FMT				
4	C					4	Y				
5	A					5	CNT				
6	B					6	+				
7	L					7	N				
8	E					8	FMT				
9	CNT					9	PNT				
0	A					127 0	FMT				
1	Z					1	FMT				
2	FMT					2	O				
3	PNT					3	P				
4	X( )					4	T				
5	3					5	•				
6	6		AZ_B			6	E				
7	↑			AZ_A		7	N				
8	9					8	T				
9	0		90	AZ_B		9	CLR				
0	X>Y					0	O				
1	1					1	—				
2	2					2	N				
3	3					3	E				
4	8					4	W				
5	4					5	CNT				
6	5					6	P				
7	0		450	AZ_A		7	R				
123 8	X⇄Y		AZ_A	450 or 90		8	O				
9	—			γ		9	B				
0	↓		γ			0	—				
1	↑		γ	γ		1	H				
2	SIN X		SIN γ	γ		2	A				
3	X⇄Y		γ	SIN γ		3	V				
4	COS X		COS γ	SIN γ		4	E				
5	↑			COS γ	SIN γ	5	CLR				
6	X( )					6	CNT				
7	3					7	7				
8	5		L	COS γ	SIN γ	8	7				
9	X		L	X	SIN γ	9	*				

STEP	KEY	CODE	X	Y	Z	STEP	KEY	CODE	X	Y	Z
130 0	O					0	X=Y				
1	O					1	1				
2	7					2	3				
3	CNT					3	8				
4	C					4	0				
5	A					5	Go To				
6	R					6	SUB/R				
7	D					7	1				
8	S					8	4				
9	CLR					9	4				
0	CNT					0	0				
1	R					1	0		0		
2	E					2	X→				
3	A					3	0				
4	D					4	X→				
5	Y					5	1				
6	CLR					6	0				
7	3					7	7				
8	—					8	1		1		
9	H					9	X→				
0	O					0	b				
1	L					1	Go To				
2	D					2	1				
3	CNT					3	CNT				
4	C					4	CNT				
5	•					5	CNT				
6	L					6	CNT				
7	E					7	CNT				
8	N					1 8	CNT				
9	G					9	CNT				
0	T					138 0	0		0		
1	H					1	X→				
2	CLR					2	a				
3	CNT					138 3	X→				
4	C					4	IND				
5	H					5	a				
6	G					6	1		1		
7	•					7	X→				
8	W					8	+				
9	I					9	a				
0	N					0	7				
1	D					1	5		75	-	
2	S					2	↑			75	
3	FMT					3	a		Reg No	75	
4	STOP		ON			4	X=Y				
5	PNT					5	1				
6	X→					6	4				
7	8					7	0				
8	↑			ON		8	5				
9	O		0	ON		9	0		0		

STEP	KEY	CODE	X	Y	Z
140 0	GoTo				
1	1				
2	3				
3	8				
4	3				
140 5	1				
6	0				
7	1		101		
8	X→				
9	a				
141 0	0		0		
1	X→				
2	IND				
3	a				
4	1		1		
5	X→				
6	+				
7	a				
8	1				
9	0				
0	9		109		
1	↑			109	
2	a		RegNo	109	
3	X=Y				
4	1				
5	4				
6	3				
7	3				
8	GoTo				
9	1				
0	4				
1	1				
2	0				
143 3	FMT				
4	GoTo				
5	CNT				
6	CNT				
7	CNT				
8	CNT				
9	CNT				
144 0	2				
1	1		21		
2	X→				
3	a				
144 4	0		0		
5	X→				
6	IND				
7	a				
8	1		1		
9	X→				

STEP	KEY	CODE	X	Y	Z
0	+				
1	a				
2	2				
3	9		29		
4	↑			29	
5	a		RegNo	29	
6	X=Y				
7	1				
8	4				
9	6				
0	6				
1	GoTo				
2	1				
3	4				
4	4				
5	4				
146 6	X( )				
7	7		K RED		
8	↑			K RED	
9	1				
0	0		10	K RED	
1	X>Y				
2	1				
3	4				
4	9				
5	3				
6	X→				
7	X				
8	6				
9	CNT				
0	CNT				
1	CNT				
2	X→				
3	X				
4	9				
5	X→				
6	X				
7	0				
8	1				
9	0				
0	0		0		
1	X→				
2	7				
149 3	SUB/R				
4	CNT				
5	CNT				
6	CNT				
7	CNT				
8	CNT				
9	CNT				

STEP	KEY	CODE	X	Y	Z
150 0	LABEL				
1	π				
2	↑			β_1	
3	X( )				
4	2		θ	β_1	
5	↑			θ	β_1
6	7				
7	5		75	θ	
8	X>Y				
9	1				
0	5				
1	2				
2	9				
3	X( )				
4	2				
5	0		Tcost+Du	θ	
6	X⇄Y		θ	Tcost+Du	β_1
7	0		0	Tcost+Du	β_1
8	X<Y				
9	1				
0	5				
1	2				
2	9				
3	1				
4	8				
5	0		180		β_1
6	R↑		β_1	180	
7	+			β_1	
8	↑				β_1
152 9	R↑		β_1		
0	GoTo				
1	6				
2	5				
3	9				
153 4	b		NW		
5	↑			n*	
6	1		1	n	
7	+			n'	
8	3		3	n'	
9	×			$3n'$	
0	3				
1	6		36	$3n'$	
2	+			$3n'+36$	
3	y→				
4	a				
5	X( )				
6	IND				
7	a		Z_{n+1}		
8	↑			Z_{n+1}	
9	X( )				

*n = NW HERE FOR CLARITY

STEP	KEY	CODE	X	Y	Z
0	5		Z	Z_{n+1}	
1	X>Y				
2	1				
3	5				
4	6				
5	5				
6	1		1		
7	X→				
8	+				
9	b				
0	GoTo				
1	1				
2	5				
3	3				
4	4				
156 5	3		3	Z_{n+1}	
6	X→				
7	−				
8	a				
9	X( )				
0	IND				
1	a		Z_n	Z_{n+1}	
2	X→				
3	3				
4	5				
5	X⇄Y		Z_{n+1}	Z_n	
6	−			ΔZ_W	
7	1		1		
8	X→				
9	+				
0	a				
1	X( )				
2	IND				
3	a		V_n	ΔZ_W	
4	↑			V_n	ΔZ_W
5	3				
6	X→				
7	+				
8	a				
9	X( )				
0	IND				
1	a		V_{n+1}	V_n	ΔZ_W
2	X⇄Y		V_n	V_{n+1}	
3	−			ΔV	
4	X⇄Y		ΔV	V_n	ΔZ_W
5	R↑		ΔZ_W	ΔV	V_n
6	÷			dV/dz	
7	X→				
8	3				
9	6				

STEP	KEY	CODE	X	Y	Z
160 0	X( )				
1	3				
2	5		Z_n	dV/dZ	V_n
3	R↑		V_n	Z_n	dV/dZ
4	X→				
5	3				
6	5				
7	X( )				
8	5		Z	Z_n	
9	−			ΔZ_0	dV/dZ
0	y→				
1	3				
2	7				
3	↓		ΔZ_0	dV/dZ	
4	×			ΔV_0	
5	X( )				
6	3				
7	5		V_n	ΔV_0	
8	+			V	
9	SUB/R				
162 0	1				
1	X→				
2	+				
3	a				
4	X( )				
5	IND				
6	a		AZ_{n+1}		
7	↑			AZ_{n+1}	
8	3		3		
9	X→				
0	−				
1	a				
2	X( )				
3	IND				
4	a		AZ_n	AZ_{n+1}	
5	−			ΔAZ_t	
6	1				
7	8				
8	0		180	ΔAZ_t	
9	X<Y				
0	1				
1	6				
2	5				
3	5				
4	CHGS		−180	ΔAZ_t	
5	X>Y				
6	1				
7	6				
8	6				
9	4				

STEP	KEY	CODE	X	Y	Z
0	GoTo				
1	1				
2	6				
3	6				
4	8				
165 5	3				
6	6				
7	0		360	ΔAZ_t	
8	−			ΔAZ	
9	GoTo				
0	1				
1	6				
2	6				
3	8				
166 4	3				
5	6				
6	0		360	ΔAZ_t	
7	+			ΔAZ	
166 8	X( )				
9	3				
0	6		ΔZ_W	ΔAZ	
1	÷			dAZ/dZ	
2	X( )				
3	3				
4	7		ΔZ_0		
5	×			ΔAZ_0	
6	X( )				
7	IND				
8	a		AZ_n	ΔAZ_0	
9	+			AZ_t	
0	GoTo				
1	SUB/R				
2	1				
3	6				
4	8				
5	7				
6	SUB/R				
168 7	0		0	Angle	
8	X>Y				
9	1				
0	6				
1	9				
2	8				
3	GoTo				
4	1				
5	7				
6	0				
7	2				
169 8	3				
9	6				

STEP	KEY	CODE	X	Y	Z
170 0	0		360	Angle	
1	+			Angle'	
170 2	3				
3	6				
4	0		360	Angle	
5	X>Y				
6	1				
7	7				
8	1				
9	1				
0	—		360°	Angle'	
171 1	SUB/R				
171 2	1		1	π	R
3	R↑		R	1	π
4	X>Y				
5	1				
6	7				
7	4				
8	1				
9	↑		R	R	1
0	1				
1	0				
2	•				
3	9		10.9	R	1
4	X⇌Y		R	10.9	
5	÷			10.9/R	
6	TAB				
7	4		LOG R	10.9/R	
8	↑			LOG R	10.9/R
9	•				
0	8				
1	7		.87		
2	X⇌Y		LOG R	.87	
3	—			.87-LR	
4	↓		.87-LR	10.9/R	
5	÷			CD	
6	GoTo				
7	1				
8	8				
9	7				
0	6				
174 1	↑			R	1
2	9		9	R	1
3	X<Y				
4	1				
5	7				
6	5				
7	4				
8	1		1=ηCD	R	1
9	GoTo				

STEP	KEY	CODE	X	Y	Z
0	1				
1	8				
2	4				
3	1				
175 4	9				
5	0				
6	0		900	R	
7	X<Y				
8	1				
9	7				
0	6				
1	8				
2	2		2=ηCD	R	
3	GoTo				
4	1				
5	8				
6	4				
7	1				
176 8	4				
9	5				
0	0				
1	0		4500	R	
2	X<Y				
3	1				
4	7				
5	8				
6	3				
7	3		3=ηCD	R	
8	GoTo				
9	1				
0	8				
1	4				
2	1				
178 3	9				
4	ENTE				
5	3		9000	R	
6	X<Y				
7	1				
8	7				
9	9				
0	7				
1	4		4=ηCD	R	
2	GoTo				
3	1				
4	8				
5	4				
6	1				
179 7	4				
8	ENT E				
9	4		40000	R	

STEP	KEY	CODE	X	Y	Z
180 0	X<Y		40000	R	
1	1				
2	8				
3	1				
4	1				
5	5		5=ncd	R	
6	GoTo				
7	1				
8	8				
9	4				
0	1				
181 1	5				
2	ENTE				
3	4		50000	R	
4	X<Y				
5	1				
6	8				
7	2				
8	5				
9	6		6=ncd	R	
0	GoTo				
1	1				
2	8				
3	4				
4	1				
182 5	2				
6	5				
7	ENTE				
8	4		250 000	R	
9	X<Y				
0	1				
1	8				
2	4				
3	0				
4	7		7=ncd	R	
5	GoTo				
6	1				
7	8				
8	4				
9	1				
184 0	8		8=ncd	R	
184 1	↑			ncd	R
2	3		3	ncd	
3	X			3ncd	
4	7				
5	4		74	3ncd	
6	+			3n+74	R
7	Y→				
8	a				
9	X( )				

STEP	KEY	CODE	X	Y	Z
0	IND				
1	a		R_B		R
2	TAB				
3	4		LOG R_B		
4	X⇄Y			LOG R_B	R
5	2		2		
6	X→				
7	+				
8	a				
9	R↑		R	2	LOG R_B
0	TAB				
1	4		LOG R		
2	R↑		LOG R_B	LOG R	
3	−			LR-LR_B	
4	X( )				
5	IND				
6	a		K_R	LR-LR_B	
7	×			K_R( )	
8	1		1		
9	X→				
0	−				
1	a				
2	X( )				
3	IND				
4	a		C_{DB}	K_R( )	
5	+			C_D	
187 6	SUB/R				
187 7	X( )				
8	1				
9	0				
0	7		C		
1	↑			C	
2	1		1	C	
3	X<Y				
4	1				
5	9				
6	9				
7	5				
8	X( )				
9	1				
0	0				
1	3		ΔZ	C=0 or 1	
2	×			0 or ΔZ	
3	X( )				
4	3		Z_B	0 or ΔZ	
5	X⇄Y		0 or ΔZ	Z_B	
6	−			Z_{Bo}	
7	X( )				
8	1				
9	0				

STEP	KEY	CODE	X	Y	Z	STEP	KEY	CODE	X	Y	Z
190 0	1		□Z	Z_{B0}		0	4				
1	—			Z_B^*		1	4				
2	y→					2	0				
3	3					3	GoTo				
4	y→					4	2				
5	5					5	5				
6	X( )					6	8				
7	1					195 7	FMT				
8	0					8	FMT				
9	5		F_T			9	P				
0	X→					0	R				
1	1					1	O				
2	X( )					2	B				
3	1					3	•				
4	0					4	H				
5	8		θ			5	A				
6	X→					6	S				
7	2					7	CNT				
8	CNT					8	N				
9	0		0			9	O				
0	X→					0	CNT				
1	1					1	S				
2	0					2	O				
3	6					3	L				
4	X→					4	•				
5	1					5	CLR				
6	0					6	FMT				
7	7					7	X( )				
8	CNT					8	1				
9	CNT					9	0				
0	CNT					0	1		□Z		
1	CNT					1	↑			□Z	
2	CNT					2	1				
3	CNT					3	0		10	□Z	
4	CNT					4	X			HT	
5	CNT					5	X( )				
6	CNT					6	4		Z_s	HT	
7	CNT					7	+			Z_{B0}	
8	CNT					8	y→				
9	CNT					9	3				
0	X( )					0	GoTo				
1	4		Z_s	Z_B^*		1	1				
2	X>Y					2	2				
3	1					3	7				
4	9					4	0				
5	5					5	X( )				
6	7					6	0		N		
7	GoTo					7	↑			N	
8	SUBR					8	1		1	N	
9	1					9	X=Y				

[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
2020 0	1					0					
1	9					1					
2	5					2					
3	7					3					
4	GOTO					4					
5	8					5					
6	6					6					
7	7					7					
8	END					8					
9						9					
0						0					
1						1					
2						2					
3						3					
4						4					
5						5					
6						6					
7						7					
8						8					
9						9					
0						0					
1						1					
2						2					
3						3					
4						4					
5						5					
6						6					
7						7					
8						8					
9						9					
0						0					
1						1					
2						2					
3						3					
4						4					
5						5					
6						6					
7						7					
8						8					
9						9					
0						0					
1						1					
2						2					
3						3					
4						4					
5						5					
6						6					
7						7					
8						8					
9						9					

Storage Registers

STORAGE		
b	NW [illegible]	
a	IND. ADDR. [illegible]	
000	R. CYCLE COUNT	
001	$F_T \to T$	
002	Θ	
003	Z_B	
004	Z_S	
005	Z	
006	K	
007	K Red. 0/10	
008	ON OUT. [illegible]	
009	W	
010	A	
011	D	
012	π or C_D	
013	V	
014	AZ	
015	α	
016	V_N	
017	D_W	
018	D_H	
019		
020	$T\cos\theta + D_L$	
021	H	SUMMARIES
022	I	SUMMARIES
023	J	SUMMARIES
024	ΣK	SUMMARIES
025	ΣW	SUMMARIES
026	ΣDW	SUMMARIES
027	$\Sigma \beta + \beta_B$	SUMMARIES
028	$\Sigma \beta$	SUMMARIES
029	$\cos\phi$	
030	β_B	
031	D_T	
032	ρ	
033	μ	
034	—	
035	Temp	
036	Temp	
037	Temp	
038	Temp	
039	Z_1	

040	V_1	WIND FIELD
041	AZ_1	WIND FIELD
042	Z_2	WIND FIELD
043	V_2	WIND FIELD
044	AZ_2	WIND FIELD
045	Z_3	WIND FIELD
046	V_3	WIND FIELD
047	AZ_3	WIND FIELD
048	Z_4	WIND FIELD
049	V_4	WIND FIELD
050	AZ_4	WIND FIELD
051	Z_5	WIND FIELD
052	V_5	WIND FIELD
053	AZ_5	WIND FIELD
054	Z_6	WIND FIELD
055	V_6	WIND FIELD
056	AZ_6	WIND FIELD
057	Z_7	WIND FIELD
058	V_7	WIND FIELD
059	AZ_7	WIND FIELD
060	Z_8	WIND FIELD
061	V_8	WIND FIELD
062	AZ_8	WIND FIELD
063	Z_9	WIND FIELD
064	V_9	WIND FIELD
065	AZ_9	WIND FIELD
066	Z_{10}	WIND FIELD
067	V_{10}	WIND FIELD
068	AZ_{10}	WIND FIELD
069	Z_{11}	WIND FIELD
070	V_{11}	WIND FIELD
071	AZ_{11}	WIND FIELD
072	Z_{12}	WIND FIELD
073	V_{12}	WIND FIELD
074	AZ_{12}	WIND FIELD
075	a_1	
076	a_0	
077	R_B	1
078	C_{DB}	1
079	K_R	1

080	R_B	2	LOADED FROM DATA CARD
081	C_{DB}	2	LOADED FROM DATA CARD
082	K_R	2	LOADED FROM DATA CARD
083	R_B	3	LOADED FROM DATA CARD
084	C_{DB}	3	LOADED FROM DATA CARD
085	K_R	3	LOADED FROM DATA CARD
086	R_B	4	LOADED FROM DATA CARD
087	C_{DB}	4	LOADED FROM DATA CARD
088	K_R	4	LOADED FROM DATA CARD
089	R_B	5	LOADED FROM DATA CARD
090	C_{DB}	5	LOADED FROM DATA CARD
091	K_R	5	LOADED FROM DATA CARD
092	R_B	6	LOADED FROM DATA CARD
093	C_{DB}	6	LOADED FROM DATA CARD
094	K_R	6	LOADED FROM DATA CARD
095	R_B	7	LOADED FROM DATA CARD
096	C_{DB}	7	LOADED FROM DATA CARD
097	K_R	7	LOADED FROM DATA CARD
098	R_B	8	LOADED FROM DATA CARD
099	C_{DB}	8	LOADED FROM DATA CARD
100	K_R	8	LOADED FROM DATA CARD
101	$\square Z$		SPEC. 77.007B
102	ΣK ORIG		SPEC. 77.007B
103	ΔZ		SPEC. 77.007B
104	V at Z_B		SPEC. 77.007B
105	F_T		SPEC. 77.007B
106	ΣK PREV		SPEC. 77.007B
107	C-COUNTER		SPEC. 77.007B
108	Θ		SPEC. 77.007B

3.4.7 SAMPLE INPUT/OUTPUT PRINT

The following are copies of the HP printed tape for a problem following Test No. 3 in Section 3.3.7.

```
           3.000*
PROG#77.007B
CONST.CABLE
LENGTH
TEST #
           3.100*
ORIG.COND
       14000.000
        4000.000
           3.142
           0.280
          25.000
         500.000
       10050.000
EST.NEW FT/ANGLE
        3400.000
          79.000
NEW WINDS
       14500.000
          50.000
         180.000

       12500.000
          60.000
         200.000

       10500.000
          65.000
         235.000

        9000.000
          55.000
         260.000

        4000.000
          35.000
         232.000
```

```
TRIAL ALT
       14000.000
       11500.000

       14200.000
       11750.000

       14000.000
       11500.000

       13800.000
       11250.000

       13600.000
       11000.000

       13400.000
       10700.000

       13200.000
       10450.000

       13000.000
       10200.000

       12800.000
        9900.000

       12980.000
       10150.000

       12960.000
       10150.000

       12940.000
       10100.000

       12920.000
       10100.000

       12900.000
```

```
ON SURF.W.APPROX
SAME LENGTH OF
CABLE OUT

BLN.ALT
       12900.000
B.WIND
          58.000
B.AZ
         196.000
S.ALT
        4026.110
HT
        8873.890
TENS
        3217.749
C.EL.ANG
          51.368
LENGTH
       10050.000
WT
         251.250
V.D
         572.656
H,I,L
        4126.115
        1635.081
        4438.278
AZ.TO BLN
          37.617
B.EL.ANG
          63.428
S.R
        9921.907
CABLE AZ
          45.084
X +E
        2709.052
Y +N
        3515.587
OPT.ENT
0-NEW PROB-HAVE
 77.007 CARDS
 READY
3-HOLD C.LENGTH
 CHG.WINDS
```

The problem shown above began with the entry of 3 at the OPT ENT STOP in Test 3 B, Section 3.3.7. A Test No. = 3.1 was assigned. Note that a cable length = 10,050 ft, the last item printed under "original conditions", is the same as the length computed in the original problem in Section 3.3.7. Values of F_T = 3400 lbs and θ = 79.0° were estimated values for the new balloon condition.

The first wind point introduced here is at an altitude of 14,500 ft or 500 ft higher than the original. Since the estimated balloon altitude is predicted to be lower than the original, a 200 ft increase would have been the minimum allowable. The new winds introduced are higher in magnitude and have less rotation than the original case. The higher magnitude affected the choice of values for F_T and θ above.

A series of trial altitudes, first up, then progressing downward 200 ft at a time, show decreasing lengths to an altitude 12,800 ft. At that point, where the cable length is less than the original value, the altitude decrement is made to be 20 ft but proceeds downward from the previous 13,000 ft altitude. Thus 12,980, 12,960, etc, are tried until at a balloon altitude of 12,900 ft, the equality of cable length is established.

The third column shows the wind at the balloon to be 58 knots at 196°. The cable angle at the winch, 51°, is considerably lower than the 79° experienced with the balloon at 14,000 ft. Greater displacements and other differences can also be noted.

Additional procedures might now be pursued. First the value of F_T and θ could be further refined (using 76.003, 4, or 5) now that a "better-than-a-guess" altitude is found. Another runthrough with these values would modify the values of all parameters computed in the first run. When all are compatible, some of the results can be used as inputs into 77.007 for print only or 77.07P for print and plot of the complete cable conditions and geometry between the balloon and the surface.

3.5 Program No. 77.007P

This version of the three-dimensional tether-cable program provides a printed and plotted output. It requires the following hardware:

HP Model 9810A Calculator
HP No. 11210A Math ROM
HP No. 11261A Printer-Plotter ROM
HP Model 9862A Plotter

If no plot is desired or no plotter equipment available, the use of Program No. 77.007 (Section 3.3) is suggested. Program No. 77.007P, however, can be run without the plotter and with either a Printer-Alpha ROM (HP No. 11211A) or the Printer-Plotter ROM by making the following substitutions in the program listing, Section 3.5.6:

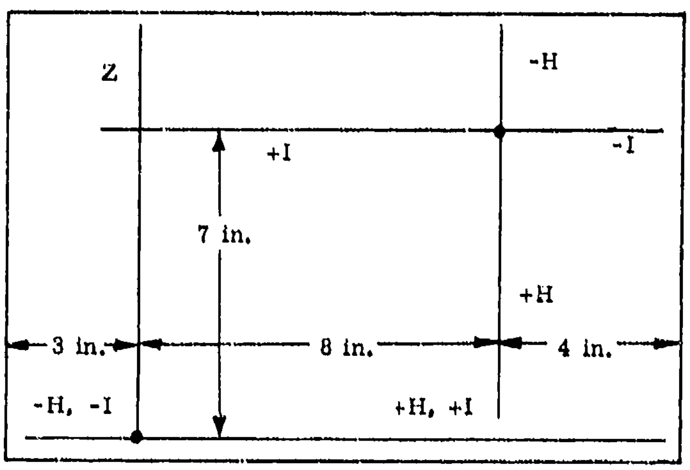
Step No.	Key
0344	CNT
0345	CNT
0346	CNT

3.5.1 PLOTTING

The use of only 2 rerun options in Program No. 77.007P provides more program steps than Program No. 77.007. This permits the plotting of the following six curves with symbols as shown on one piece of ~ 10x15 in. paper.

	Symbol	Parameters		Symbol	Parameters
(a)	+	Z-H	(d)	T	Z-T
(b)	.	Z-I	(e)	⍁	Z-θ
(c)	.	H-I	(f)	x	Z-q

Before computation and plotting of data commence, various events take place to prepare the plot paper. Axes for curves a, b, and c are drawn as follows:



In addition a fifth line is drawn along the top edge of the paper, 3-in. above the upper I-axis for the tension, d. During the drawing of axes, tick marks are added at user specified intervals. No tick-marks for the angle θ are provided; the scale for θ is always 10° per inch or 90° is 9-in. to the right of the Z-axis. No tick-marks for q are provided; the scale for q is always 2-lbs/ft^2 per inch.

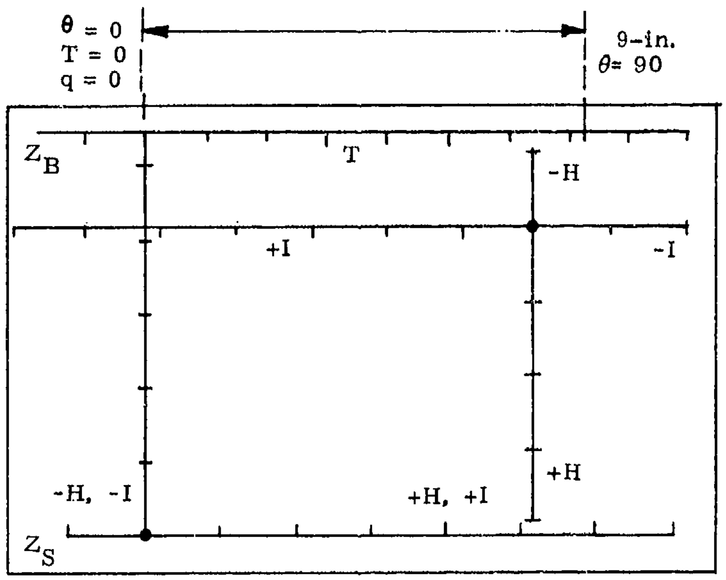
Three constants are entered at STOP 5 in order to scale the axes and set tick-mark spacing. The three constants are:

N. The interval in full-scale feet between tick-marks on the Z, H, and I axes. Z_S OR SURFACE ALTITUDE is located at the origin of the Z-H or Z-I set of axes with Z_B located at the top of the Z-axis. Since the available length of the Z-axis is 10-in., the magnitude of the altitude excursion, Z_B-Z_S, will determine the scale of the Z, H, and I axes and the tick-mark intervals. Therefore in the case of a high altitude balloon with light winds and a small amount of cable displacement, the H - I plot may be quite small.

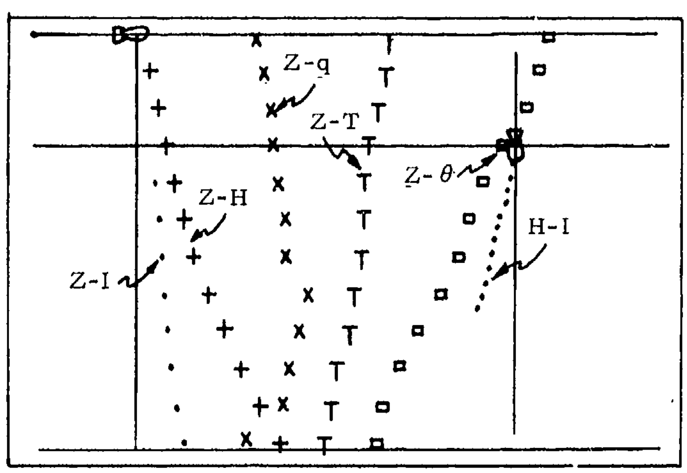
M. The interval in pounds between tick-marks on the tension axis.

P. A factor used for scaling the tension axis. The value of the balloon total force, F_T, rounded to the nearest 10^P pounds is located 6-in. to the right of the Z axis. Tick-marks are drawn at M intervals with one at the Z axis where T=0. The rounding of F_T above is done only for establishing the scale of the T axis—the computations are unaffected. If for example, F = 7075, P = 2, and M = 1000 the process would round F_T to 7100 lb and divide by 6 to get the scale factor. Using the scale factor, the 1000-lb intervals would be converted to the proper linear dimension for marking on the paper. When plotted, the exact F_T point would appear just to the right of the 7000-lb tick-mark.

The plotting paper will then appear as shown on the following page (without notations) before any computations or plotting commence; the upper axis to the left of the Z-axis represents imaginary negative values of T and θ and can be ignored.



The plotting of 5 points representing the bottom end conditions of each cable element is made after the printing of 23 parameters for each element. This print then plot cycle continues down the cable until the surface is reached. The completed plot would then appear as shown below:



The spacing of the axes favors conditions of positive H and I values although negative values of lesser magnitude can be accommodated. The H-I plot is really a X_B - Y_B plot looking down on the balloon and cable from above. The balloon, located at the origin of the H and I axes, is pointing down the paper in the positive H direction the same as in Figure 4. Similarly, the wind at the balloon is pointed up the paper. Knowing this balloon wind azimuth, it is then simple to draw a North reference line on the plot with a protractor if desired. Similarly the balloon in the vertical plane is located at the top of the Z-axis at Z_B and is pointing to the right. If a two dimensional wind problem were plotted, the Z-I curve would be hidden in the Z-axis (I=0) and the H-I curve would be hidden in the vertical H-axis (I=0).

The number of points plotted for each of the six curves will equal the number of cable elements required to reach the surface. If any plotted point should be called that exceeds the 10 × 15-in. boundaries of the paper, it will not be plotted but the other points will still be plotted. See also Section 3.5.7.

3.5.2 RERUN OPTIONS

When a problem is solved, the cable end conditions printed, and the plotter is stopped, the program is designed to offer the user a choice of methods with which to proceed to the next problem. A message is printed:

OPT.ENT (OPTION, ENTER)

followed by a choice of two numbers and a STOP.

When a completely new problem requiring reentry of different altitudes, winds, and cable parameters is next to be run, the number 0 is entered. The program will clear all but the storage registers loaded from the data cards and cycle back to the initial printing of the program number and title.

The normal runs (printed RUN#1MAXALT) provide a cable solution (a) for the balloon at a specific altitude subjected to the winds specified in the first group of wind entries and (b) for the cable subjected to a variety of wind conditions specified at the lower altitudes down to the surface. One practical and sometimes limiting problem is concerned with raising or lowering the balloon through the same specified wind field. Rather than a reentry of all variables required at each of many lower altitudes, a method is provided to simplify a series of runs at decreasing altitudes.

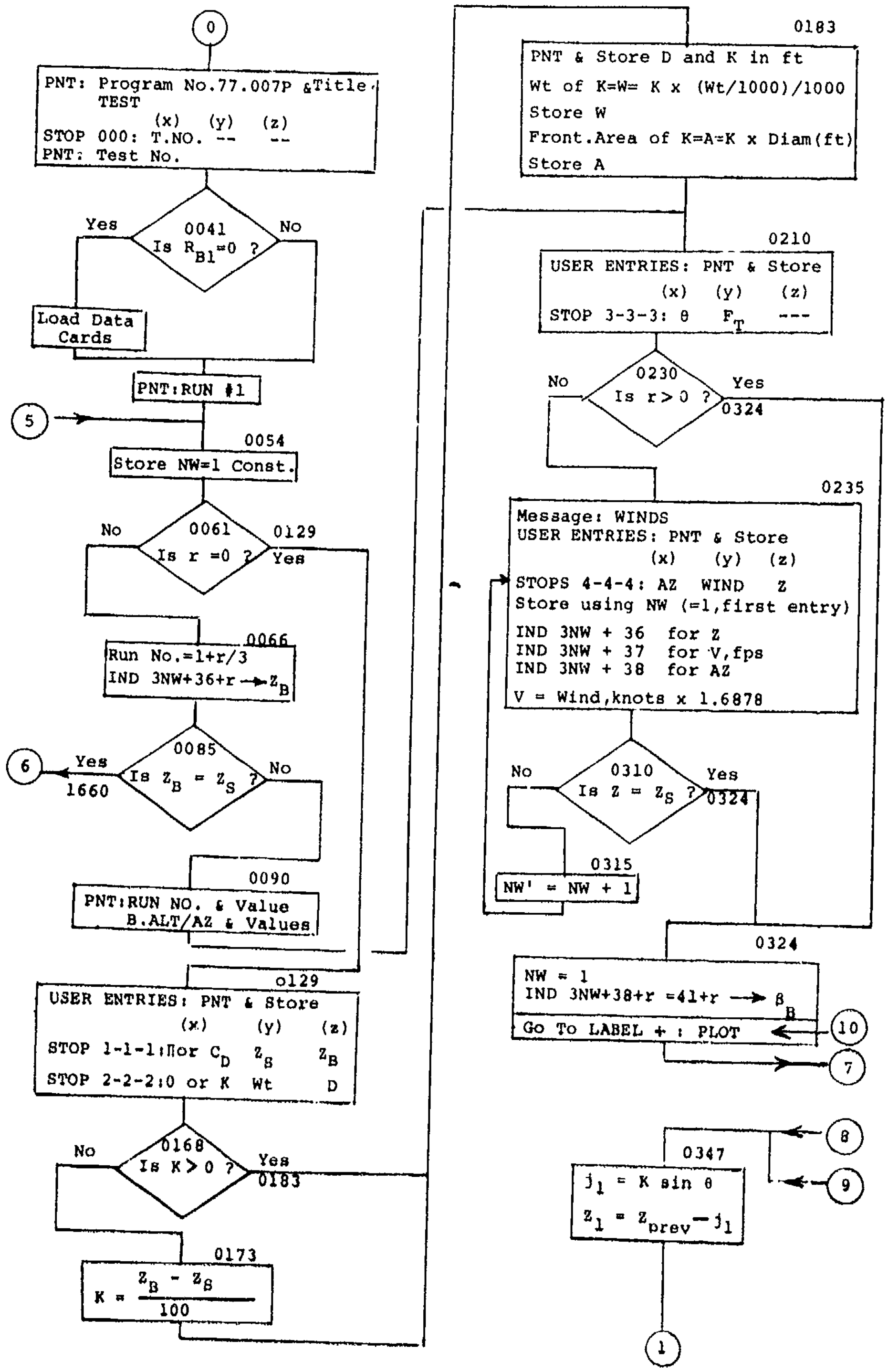
If the number 2 is entered at the above STOP, the program will clear the summation registers and retain the surface altitude, cable specifications, and wind field, and setup RUN No. 2 with the balloon at the second lower altitude in the wind field table. Because the balloon total force and angle at this new altitude will differ from the previous balloon altitude, only one STOP is needed for these two entries. A full solution is then provided for this condition. At its end, the program loops to RUN No. 3 and places the balloon at the third lower altitude in the wind field

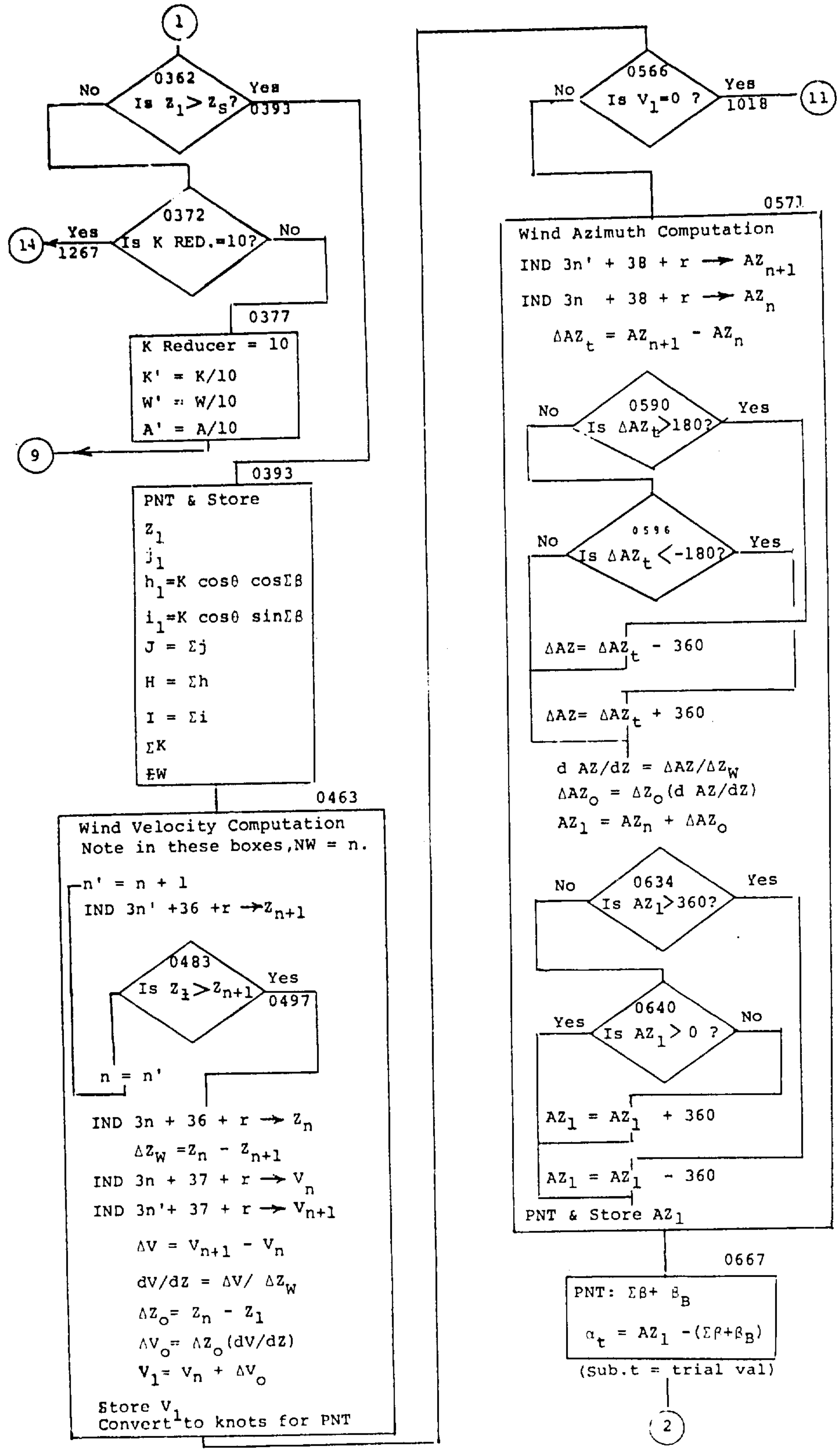
table. These automatic loops continue until the surface is reached. At that point no OPT. END message is printed. The program cycles back to a restart as if Option O were selected.

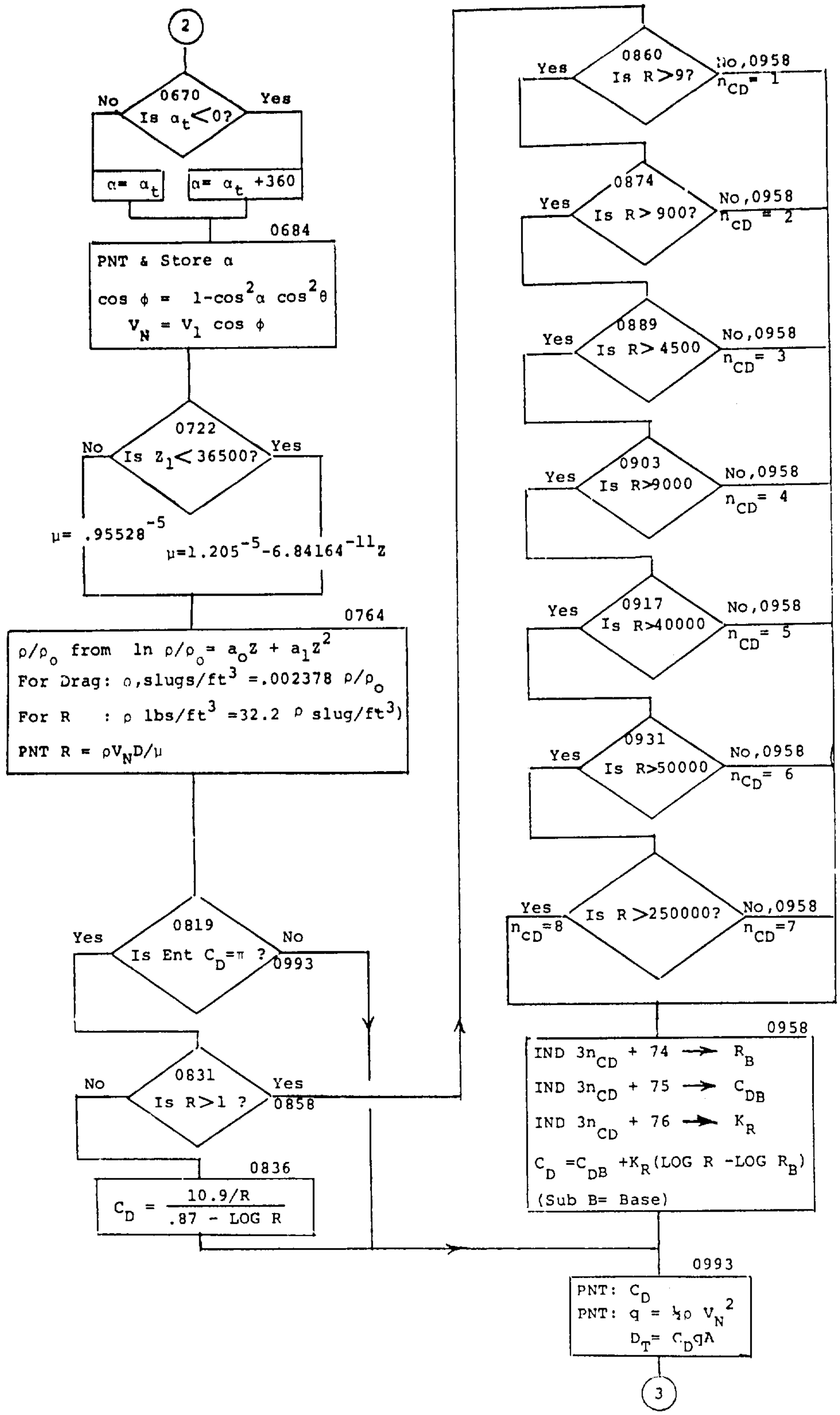
When considering the use of this lowering altitude cycle option, it is necessary to initially enter a wind field table which specifies a sufficient number of altitudes to provide a complete performance analysis. Altitudes having particularly strong winds, of concern in balloon performance, would of course be included in the initial definition of the wind field acting on the cable. Similarly, it is necessary to have the values of the balloon total force, F_T, and its angle, θ, available for entry at each altitude as indicated in the INPUT DATA FORM. The use of Program No. 76.003 or No. 76.005, Reference 1, for each of the altitudes in the wind field table will provide the values of F_T and θ.

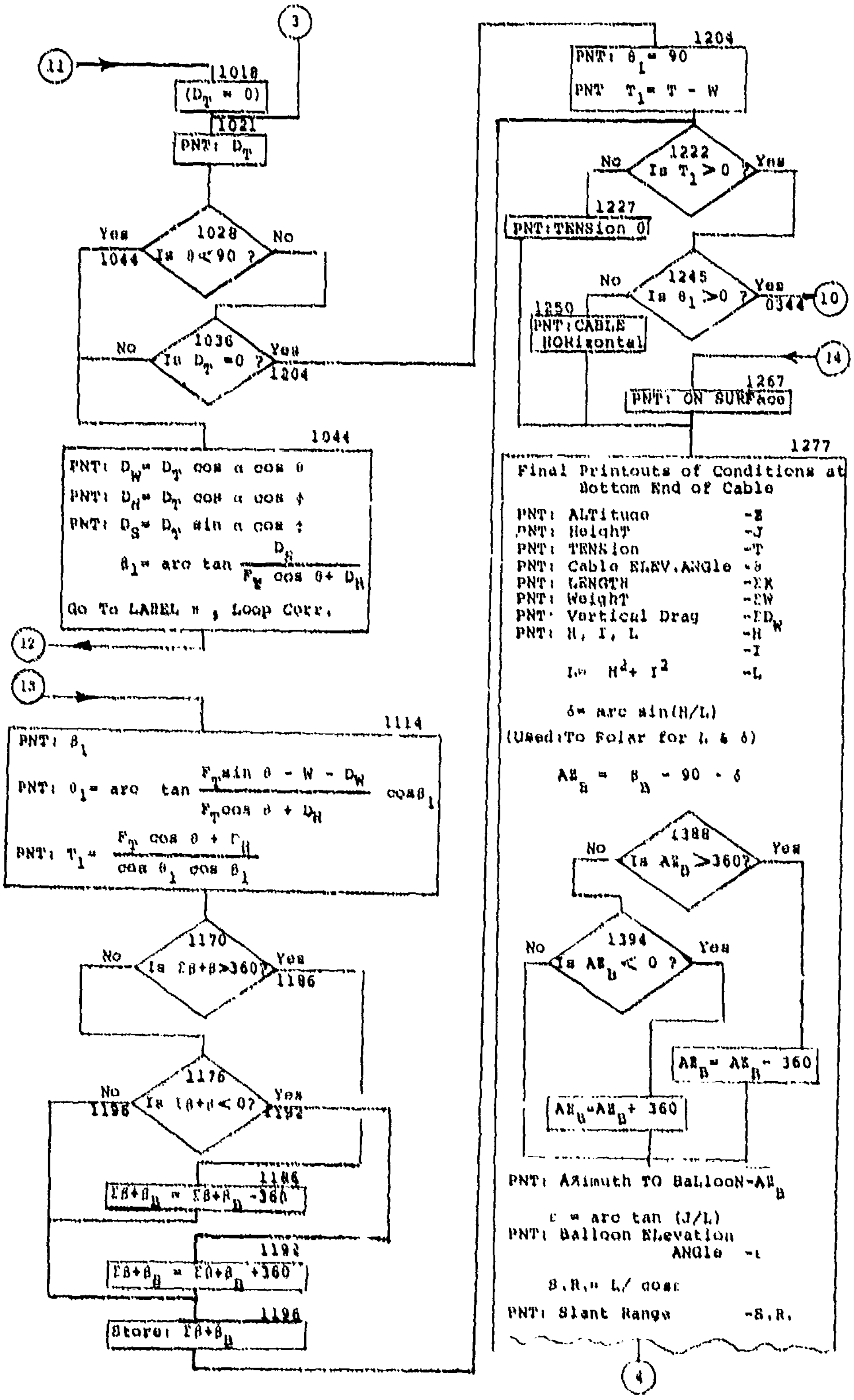
When using the plotter with the OPT 2 mode, it should be noted that no new axes or tick marks will be drawn for each of the balloon altitudes. If the original graph from the maximum balloon altitude solution is left on the plotter and only 2 or 3 lower altitude solutions were involved, pen color changes might help to keep each set identifiable. All scales except the tension scale remain constant for the lower altitude runs. The tension scale may or may not remain fixed depending on the value of the newly entered F_T at each balloon altitude. The original values of M and P entered at STOP 5 are retained and operate on the new F_T. If F_T rounds to the same number as in the RUN No. 1 for the maximum altitude, then the scale remains the same and the Z-T curve may be referenced to the tension axis and tick marks.

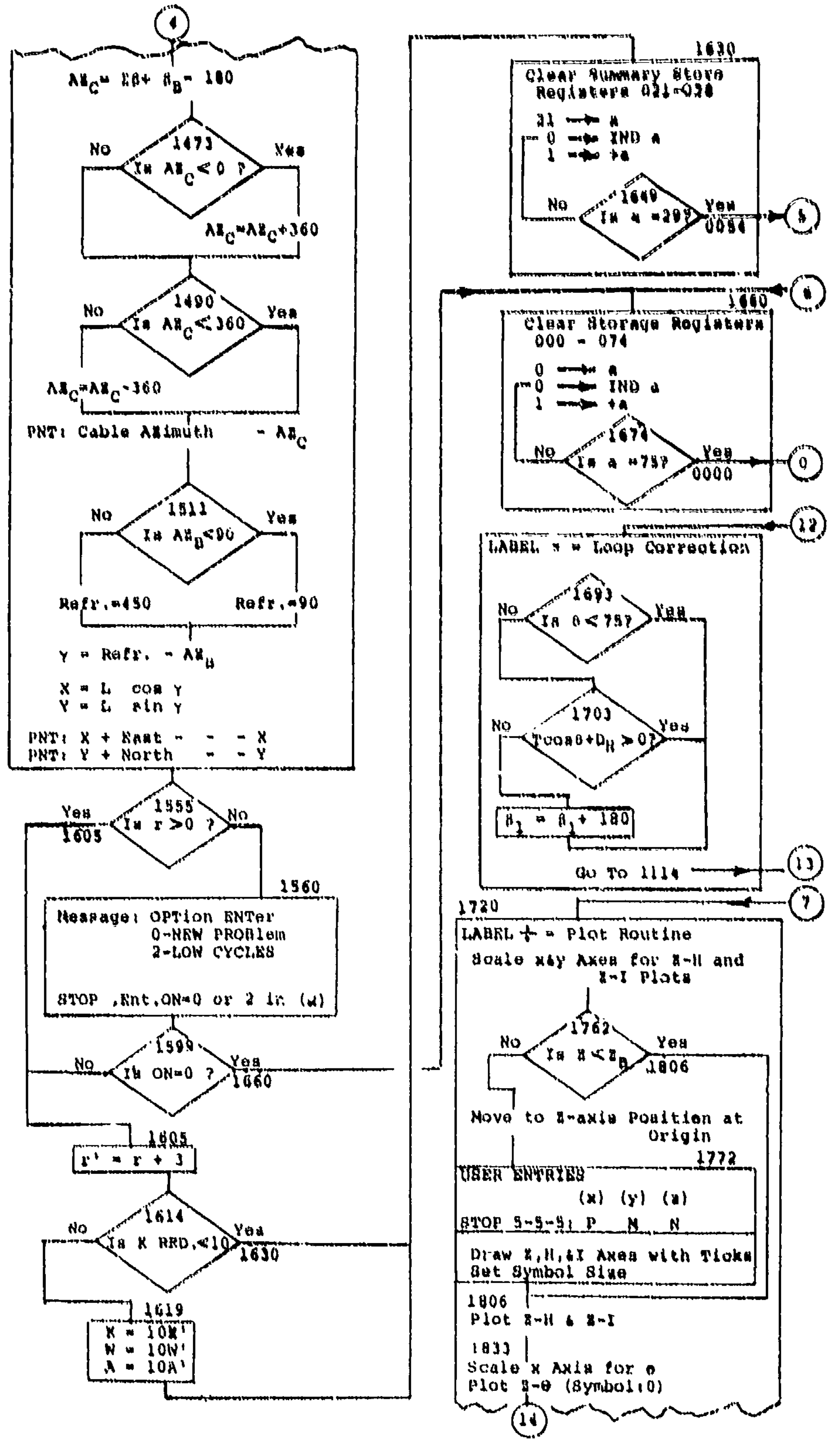
3.5.3 FLOW CHART

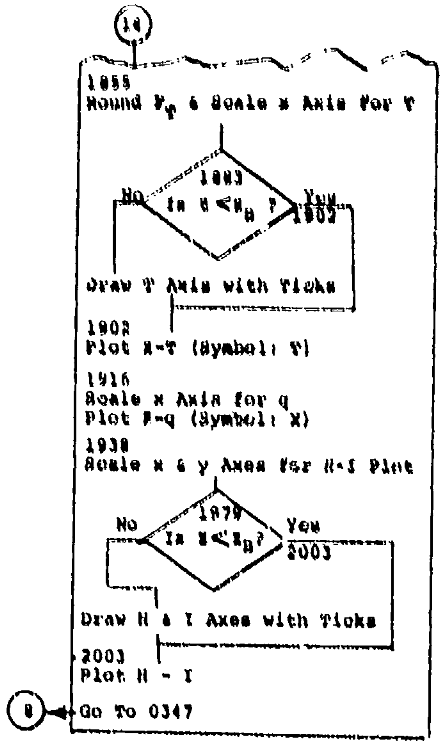



1
0362
Is $Z_1 > Z_S$?
No
Yes
0393
0372
Is K RED.=10?
Yes
1267
14
No
0377
K Reducer = 10
K' = K/10
W' = W/10
A' = A/10
9
0393
PNT & Store
Z_1
j_1
h_1=K cosθ cosΣβ
i_1=K cosθ sinΣβ
J = Σj
H = Σh
I = Σi
ΣK
ΣW
0463
Wind Velocity Computation
Note in these boxes,NW = n.
n' = n + 1
IND 3n' +36 +r → Z_{n+1}
0483
Is $Z_1 > Z_{n+1}$
Yes
0497
n = n'
IND 3n + 36 + r → Z_n
$\Delta Z_W = Z_n - Z_{n+1}$
IND 3n + 37 + r → V_n
IND 3n'+ 37 + r → V_{n+1}
$\Delta V = V_{n+1} - V_n$
$dV/dZ = \Delta V/\Delta Z_W$
$\Delta Z_o = Z_n - Z_1$
$\Delta V_o = \Delta Z_o (dV/dZ)$
$V_1 = V_n + \Delta V_o$
Store V_1
Convert to knots for PNT
0566
No
Is V_1=0 ?
Yes
1018
11
0571
Wind Azimuth Computation
IND 3n' + 38 + r → AZ_{n+1}
IND 3n + 38 + r → AZ_n
$\Delta AZ_t = AZ_{n+1} - AZ_n$
No
0590
Yes
Is $\Delta AZ_t > 180$?
No
0596
Yes
Is $\Delta AZ_t < -180$?
$\Delta AZ = \Delta AZ_t - 360$
$\Delta AZ = \Delta AZ_t + 360$
$d\,AZ/dZ = \Delta AZ/\Delta Z_W$
$\Delta AZ_o = \Delta Z_o (d\,AZ/dZ)$
$AZ_1 = AZ_n + \Delta AZ_o$
No
0634
Yes
Is $AZ_1 > 360$?
Yes
0640
No
Is $AZ_1 > 0$?
$AZ_1 = AZ_1 + 360$
$AZ_1 = AZ_1 - 360$
PNT & Store AZ_1
0667
PNT: $\Sigma\beta + \beta_B$
$\alpha_t = AZ_1 - (\Sigma\beta + \beta_B)$
(Sub.t = trial val)
2


2
0670 Is α_t < 0?
No
Yes
α = α_t
α = α_t + 360
0684
PNT & Store α
cos φ = 1-cos²α cos²θ
V_N = V_1 cos φ
0722 Is Z_1 < 36500?
No
Yes
μ = .95528^-5
μ=1.205^-5-6.84164^-11 Z
0764
ρ/ρ_o from ln ρ/ρ_o = a_o Z + a_1 Z²
For Drag: ρ, slugs/ft³ = .002378 ρ/ρ_o
For R : ρ lbs/ft³ = 32.2 ρ slug/ft³)
PNT R = ρV_N D/μ
0819 Is Ent C_D = π ?
Yes
No 0993
0831 Is R > 1 ?
No
Yes 0858
0836
C_D = (10.9/R) / (.87 - LOG R)
0860 Is R > 9?
Yes
No, 0958 n_CD = 1
0874 Is R > 900?
Yes
No, 0958 n_CD = 2
0889 Is R > 4500
Yes
No, 0958 n_CD = 3
0903 Is R > 9000
Yes
No, 0958 n_CD = 4
0917 Is R > 40000
Yes
No, 0958 n_CD = 5
0931 Is R > 50000
Yes
No, 0958 n_CD = 6
Is R > 250000?
Yes n_CD = 8
No, 0958 n_CD = 7
0958
IND 3n_CD + 74 → R_B
IND 3n_CD + 75 → C_DB
IND 3n_CD + 76 → K_R
C_D = C_DB + K_R (LOG R - LOG R_B)
(Sub B= Base)
0993
PNT: C_D
PNT: q = ½ρ V_N²
D_T = C_D qA
3


1018
$(D_T = 0)$
1021
PNT: D_T
1028
Is $\theta < 90$?
1036
Is $D_T = 0$?
1044
PNT: $D_W = D_T \cos\alpha \cos\theta$
PNT: $D_H = D_T \cos\alpha \cos\phi$
PNT: $D_S = D_T \sin\alpha \cos\phi$
$\beta_1 = \arctan \frac{D_S}{F_W \cos\theta + D_H}$
Go To LABEL n, Loop Corr.
1114
PNT: β_1
PNT: $\theta_1 = \arctan \frac{F_T \sin\theta - W - D_W}{F_T \cos\theta + D_H} \cos\beta_1$
PNT: $T_1 = \frac{F_T \cos\theta + D_H}{\cos\theta_1 \cos\beta_1}$
1170
Is $\Sigma\beta+\beta > 360$?
1176
Is $\Sigma\beta+\beta < 0$?
1186
$\Sigma\beta+\beta_B = \Sigma\beta+\beta_B - 360$
1192
$\Sigma\beta+\beta_B = \Sigma\beta+\beta_B + 360$
1196
Store: $\Sigma\beta+\beta_B$
1204
PNT: $\theta_1 = 90$
PNT $T_1 = T - W$
1222
Is $T_1 > 0$?
1227
PNT: TENSion 0
1245
Is $\theta_1 > 0$?
1250
PNT: CABLE HORizontal
1267
PNT: ON SURFace
1277
Final Printouts of Conditions at Bottom End of Cable
PNT: ALTitude -Z
PNT: HeighT -J
PNT: TENSion -T
PNT: Cable ELEV.ANGle -δ
PNT: LENGTH -ΣK
PNT: WeighT -ΣW
PNT: Vertical Drag $-\Sigma D_W$
PNT: H, I, L -H -I
$L = H^2 + I^2$ -L
$\delta = \arcsin(H/L)$
(Used: To Polar for L & δ)
$AZ_B = \beta_B - 90 - \delta$
1388
Is $AZ_B > 360$?
1394
Is $AZ_B < 0$?
$AZ_B = AZ_B - 360$
$AZ_B = AZ_B + 360$
PNT: AZimuth TO BaLlooN-AZ_B
$\epsilon = \arctan(J/L)$
PNT: Balloon ELevation ANGle $-\epsilon$
$S.R. = L/\cos\epsilon$
PNT: Slant Range -S.R.
Yes
No
11
3
12
13
10
14
4


4
$AZ_C = \Sigma\theta + \theta_B - 180$
1473
Is $AZ_C < 0$?
No
Yes
$AZ_C = AZ_C + 360$
1490
Is $AZ_C < 360$
No
Yes
$AZ_C = AZ_C - 360$
PNT: Cable Azimuth - AZ_C
1511
Is $AZ_B < 90$
No
Yes
Refr.=450
Refr.=90
γ = Refr. - AZ_B
X = L cos γ
Y = L sin γ
PNT: X + East - - - X
PNT: Y + North - - Y
1555
Is r > 0 ?
Yes
1605
No
1560
Message: OPTion ENTer
0-NEW PROBlem
2-LOW CYCLES
STOP ,Ent.ON=0 or 2 in (a)
1599
Is ON=0 ?
No
Yes
1660
1605
r' = r + 3
1614
Is K RED.<10
No
Yes
1630
1619
K = 10K'
W = 10W'
A = 10A'
1630
Clear Summary Store
Registers 021-028
21 → a
0 → IND a
1 → +a
1649
Is a =29?
No
Yes
0084
5
6
1660
Clear Storage Registers
000 - 074
0 → a
0 → IND a
1 → +a
1674
Is a =75?
No
Yes
0000
0
12
LABEL ÷ = Loop Correction
1693
Is θ < 75?
No
Yes
1703
Tcosθ+D_H > 0?
No
Yes
$\theta_1 = \theta_1 + 180$
Go To 1114
13
7
1720
LABEL ÷ = Plot Routine
Scale x&y Axes for Z-H and Z-I Plots
1762
Is Z < Z_B
No
Yes
1806
Move to Z-axis Position at Origin
USER ENTRIES
1772
(x) (y) (z)
STOP 5-5-5: P N N
Draw Z,H,&I Axes with Ticks
Set Symbol Size
1806
Plot Z-H & Z-I
1833
Scale x Axis for θ
Plot Z-θ (Symbol:0)
14


1855
Scale x Axis for q
1902
1916
Scale x Axis for q
No
Yes
2003
Draw H & I Axes with Ticks
2003
Plot H - I
Go To 0347

3.5.4 OPERATING INSTRUCTIONS AND NOTES

KEY STROKES | ENTRIES

[RUN]

[END]

[FIX] [2,] [3] ---(Number of decimal places desired)

Insert Side 1 of Program Cards

[LOAD]

Continue inserting Sides 2, 3, and 4 of Program Cards

Insert Side 1 of Data Cards

[END]

[CONT]

(Following heading is now printed)

PROG # 77.007P

3-DIM. TETHER

TEST

	(X)	(Y)	(Z)
At STOP 0-0-0, Enter :	Test No/I.D.	————	————

[CONT]

Following printing of the test number or I.D., Side 1 of the Data Card will load.

Continue inserting Sides 2 and 3 of Data Cards

Note that loading of Program and Data Cards will only be required one time until machine is turned off.

(Following is now printed)

RUN #1

At STOP 1-1-1, Enter:	π or C_D*	Z_S (ft, MSL)	Z_B (ft, MSL)

*Enter π to use built-in cylinder C_D variation or enter C_D value which will be held constant throughout program run.

[CONT]

At STOP 2-2-2, Enter:	0 or K* (Element Length)	Wt/1000 ft (lbs) (Cable)	Diam. (in.) (Cable)

*Enter 0 to set $K = \frac{Z_B - Z_S}{100}$ or enter K in ft.

[CONT]

At STOP 3-3-3, Enter:	θ (deg) (Angle of F_T to Horizon)	F_T (lb) (Balloon Tot. Force)	————

[CONT]

WINDS (Printed)

At STOP 4-4-4, Enter:	Azimuth of Wind (deg)	Wind (knots)	Z_B (ft, MSL)

This entry must be for conditions at balloon altitude.

	(X)	(Y)	(Z)
$\boxed{\text{CONT}}$			
At STOP 4-4-4, Enter:	Azimuth	Wind	Z

Stop 4's will repeat until the last set of entries for the surface condition at Z_S are inserted. A total of 12 sets of wind entries may be made including the balloon and surface conditions.

$\boxed{\text{CONT}}$

The above sets of entries will be printed out in groups when CONT is struck after STOPS 1, 2, 3, and 4. The entered or computed value will be printed in the case of K.

(If no plot is wanted, see Section 3.5)
The plotter routine next becomes operative requiring the following:
Install plot paper with limits set to 10 × 15-in.

At STOP 5-5-5, Enter:	P	M	N
	(Rounding Factor 10^P-lb, for Tension Scale)	(Tension Tick Mark Interval, lb)	(Spacial Tick Mark Interval, ft)

$\boxed{\text{CONT}}$

Axes and tick-marks will now be drawn by the plotter per Section 3.5.1. Note the scale of θ is always 10°/in. with θ = 0 at the Z-axis and the scale of q is always 2 lb/ft^2 per in., q = 0 at the Z-axis.

The program now begins computation starting at the balloon and works downward one element at a time to the surface. Twenty-three parameters are first printed then the following six points are plotted, all representing the conditions at the bottom end-point of each element. The six points are plotted with the following symbols:

+	Z-H
.	Z-I
⧄	Z-θ
T	Z-T
X	Z-q
.	H-I

Note: To avoid the printout of any or all of the twenty-three parameters shown below, replace PNT with CNT at the associated program step numbers.

Step No.		
0395	Z	Altitude, ft MSL
0397	j	Vert. Distance, top to bottom of element, ft
0420	h	Horiz. Distance, parallel to balloon axis, ft
0431	i	Horiz. Distance, perpendicular to baln, axis, ft
0439	J	Total Vert. Distance, balloon to bottom of element, ft
0443	H	Total Horiz. Dist., Σh, balloon to bottom of element, ft
0447/8	I	Total Horiz. Dist., Σi, balloon to bottom of element, ft

Step No.		
0452	ΣK	Total Element (Cable) Length, ft
0462	ΣW	Total Element (Cable) Weight, lb
0563	Wd	Wind Velocity at bottom of element, knots
0665	AZ	Wind Azimuth at bottom of element, deg
0667	$\Sigma\beta + \beta_B$	Azimuth of Vert. Plane Containing the Element, deg
0688	α	Wind Incidence Angle on the Element, deg
0812	R	Reynolds Number
0994	C_D	Drag Coefficient
1011	q	Dynamic Pressure, lb/ft^2, Based on Velocity Normal to Element
1021	D_T	Total Element Drag, lb
1060	D_W	Vertical Drag Component, lb
1072	D_H	Horiz. Drag Comp. in Vertical Plane of the Element, lb
1086/7	D_S	Horiz. Drag Comp. to Vertical Plane of the Element, lb
1114	β_1	Horiz. Rotation of Tension Vector at bottom end or Horiz. Rotation of the next element's vertical plane, deg
1148 1206	θ_1	Pitch Angle Downward of Tension Vector at bottom end or Elevation Angle of next element above the horizon, deg
1161/2 1216/7	T_1	Tension at bottom end or at top end of next element, lb

Groups of the values of the above 23 parameters will continue to be printed for points down the cable until one of the following conditions is encountered:

(a) The cable reaches the earth's surface at Z_S—the winch location,

(b) The tension becomes zero,

(c) The cable becomes horizontal.

(In (a), the computational techniques used do not yield a precise Z_S condition. The final Z will be higher than Z_S by an amount less than K sin θ/10, usually no more than a few feet.)

The final printout includes the abbreviated names and values of the following parameters. They describe the conditions at the winch if condition (a) is attained or at the cable lower end which is above the surface if conditions (b) or (c) are indicated.

(a) ON SURFACE or (b) TENSION. 0 or (c) CABLE HOR

ALT	Z	Altitude, ft
HT	J	Vertical Height, ft
TENS	T	Cable Tension, lb
C. ELEV. ANG	θ	Elevation Angle of Cable above Horizon, deg
LENGTH	ΣK	Cable Length, ft
WT	ΣW	Cable Weight, lb
V. D	ΣD_W	Total Vertical Drag Component, lb

H, I, L	H	Tot. Horiz. Distance along Y_B or Y_W axis, ft
	I	Tot. Horiz. Distance along X_E or X_W axis, ft
	L	Min. Direct Horizontal Dist. to Balloon, ft
AZ. TO BLN	AZ_B	Azimuth Angle to Balloon, deg
B. EL. ANG	ϵ	Elevation Angle to Balloon, deg
S. R	SR	Slant Range to Balloon, ft
C. AZ	AZ_C	Azimuth Angle of Cable (Out of Winch), deg
X + E	X	X Coordinate to Balloon, ft
Y + N	Y	Y Coordinate to Balloon, ft

At this point the initial problem entered is solved with printing and plot completed. If this was an initial run (RUN #1 MAX ALT), the following is printed and STOP provided to permit 2 optional ways to rerun the program.

OPT. ENT
0 -NEW PROB
2 -LOW CYCLES

At STOP , Enter: 0 or 2 in (X)
$\boxed{\text{CONT}}$

If 0 is Entered - New Problem—Use for completely new problem. All but the permanent storage registers containing density and drag coefficient constants will be cleared, program returns to start with reprint of Number and Title.

If 2 is Entered - Lower Altitude Cycles—Use when analysis is desired with the balloon lowered to each of the altitudes specified in the wind field table. Summary storage registers are cleared and the program returns to start with reprint of Number and Title. Instead of RUN#1 the following is printed:

RUN #
2
B. ALT/AZ
Z_B
β_B

Program then goes to STOP 3 for entry of balloon total force and angle at this new altitude Z_B which is the second altitude originally entered in the wind table. The program will then make a complete solution to the surface (or to zero tension or horizontal cable) for this new balloon altitude. However at the end of RUN #2, no option is provided since the program cycles on to the third altitude point in the wind field table, sets up RUN #3 and goes to STOP 3 again for entry of the two balloon parameters at this new altitude. Thus a solution is provided for each altitude in the wind field table except for Z_S. When Z_S is detected, the program will terminate the same as if 0 were entered at OPT ENT and return to the start with a reprint of Number and Title. Refer to Section 3.5.1, for plotting notes with this OPT 2.

Note No. 1 - If incorrect data is believed to have been entered, do not press STOP END to restart program. For correct and safe clearing of registers press following:

STOP

GO TO

1

5

6

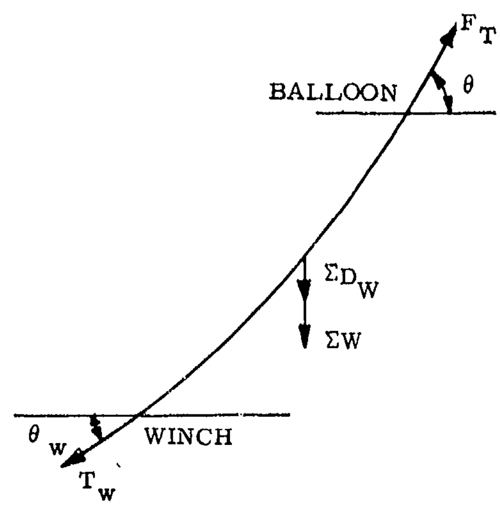
0

After OPT ENT message is printed, enter 0, then:

CONT

Note No. 2 - Proof of solutions can be made by summation of the vertical forces. Summary of the horizontal forces is not possible because summation along two fixed horizontal axes could not be handled within the available space.

Therefore: $F_T \sin\theta = T_W \sin\theta_W + \Sigma W + \Sigma D_W$



3.5.5 INPUT DATA FORM

Test No.:________ Date:______________ Notes:______________________________

INPUT				77.007P
STOP NO.	ITEM		VALUE	
1-1-1	MAX. BALLOON ALTITUDE	Z_B		Ft. MSL
	SURFACE ALTITUDE	Z_S		Ft. MSL
	CABLE C_D or π for Internal Comp.			
2-2-2	CABLE DIAMETER	D		Inches
	CABLE WEIGHT per 1000 ft			Lb.
	CABLE ELEMENT LENGTH, K OR 0 for $K = (Z_B - Z_S)/100$			Ft.
3-3-3	BALLOON TOTAL FORCE	F_T		Lb.
	ANGLE OF TOTAL FORCE	θ		Deg.
4-4-4	WIND PROFILE			

		For Stop 3 Opt. Lower Alt. Cycles				For Stop 3 Opt. Lower Alt. Cycles	
1. Z_B	Ft. MSL			7. Z	Ft. MSL	F_T	Lb.
Wind	Knots			Wind	Knots	θ	Deg.
AZ	Deg.			AZ	Deg.		
2. Z	Ft. MSL	F_T	Lb.	8. Z	Ft. MSL	F_T	Lb.
Wind	Knots	θ	Deg.	Wind	Knots	θ	Deg.
AZ	Deg.			AZ	Deg.		
3. Z	Ft. MSL	F_T	Lb.	9. Z	Ft. MSL	F_T	Lb.
Wind	Knots	θ	Deg.	Wind	Knots	θ	Deg.
AZ	Deg.			AZ	Deg.		
4. Z	Ft. MSL	F_T	Lb.	10. Z	Ft. MSL	F_T	Lb.
Wind	Knots	θ	Deg.	Wind	Knots	θ	Deg.
AZ	Deg			AZ	Deg.		
5. Z	Ft. MSL	F_T	Lb.	11. Z	Ft. MSL	F_T	Lb.
Wind	Knots	θ	Deg.	Wind	Knots	θ	Deg.
AZ	Deg.			AZ	Deg.		
6. Z	Ft. MSL	F_T	Lb.	12. Z	Ft. MSL		
Wind	Knots	θ	Deg.	Wind	Knots		
AZ	Deg.			AZ	Deg.		

A minimum of two wind points must be specified. Conditions at Z_B must be the first point. Conditions at Z_S must be the last point. A maximum of twelve wind points may be specified.

5-5-5	PLOT CONSTANTS Spacial Tick-Mark Intervals, N Tension Tick-Mark Intervals, M Tension Rounding Factor, P		 Ft. Ft.

3.5.6 PROGRAM NO. 77.007P

STEP	KEY	CODE	x	y	z	STEP	KEY	CODE	x	y	z
000 0	CLR					005 0	N				
1	FMT					1	#				
2	FMT					2	1				
3	P					3	FMT				
4	R					005 4	1		NW=1		
5	O					5	X→				
6	G					6	b				
7	#					7	0		0		
8	7					8	↑			0	
9	7					9	X()				
0	•					0	0		r	0	
1	0					1	X=Y				
2	0					2	0				
3	7					3	1				
4	P					4	2				
5	CLR					5	9				
6	3					6	↑			r	
7	-					7	3		3	r	
8	D					8	÷			r/3	
9	1					9	1		1	r/3	
0	M					0	+			RUN No	
1	•					1	X()				
2	T					2	0		r		
3	E					3	↑			r	RUN No
4	T					4	3				
5	H					5	9		39	r	RUN No
6	E					6	+		3NW+36+r		
7	R					7	y→				
8	CLR					8	a				
9	T					9	X()				
0	E					0	IND				
1	S					1	a		Z_B		
2	T					2	X⇌Y			Z_B	
3	FMT					3	X()				
4	STOP		T. No.			4	4		Z_S	Z_B	RUN No
5	PNT					5	X=Y				
6	X()					6	1				
7	7					7	6				
8	7		R_{B1}			8	6				
9	↑					9	0				
0	0		0	R_{B1}		0	y→				
1	X=Y					1	5				
2	FMT					2	R↑		RUN No	Z_S	Z_B
3	X()					3	FMT				
4	CNT					4	FMT				
5	CNT					5	R				
6	FMT					6	U				
7	FMT					7	N				
8	R					8	#				
9	U					9	FMT				

STEP	KEY	CODE	x	y	z
010 0	PNT		RUN No	Z_S	Z_B
1	2				
2	X→				
3	+				
4	a				
5	X( )				
6	IND				
7	a		B_B	Z_S	Z_B
8	R↑		Z_B	B_B	Z_S
9	FMT				
0	FMT				
1	B				
2	•				
3	A				
4	L				
5	T				
6	÷				
7	A				
8	Z				
9	FMT				
0	PNT		Z_B		
1	↓		B_B		
2	PNT		B_B		
3	PNT				
4	Go To				
5	0				
6	2				
7	1				
8	0				
012 9	1		1		
0	↑		1	1	
1	↑		1	1	1
2	STOP		π or C_D	Z_S	Z_B
3	R↑		Z_B	π or C_D	Z_S
4	PNT		Z_B		
5	X→				
6	3				
7	X→				
8	5				
9	R↑		Z_S	Z_B	π or C_D
0	PNT		Z_S		
1	X→				
2	4				
3	R↑		π or C_D	Z_S	Z_B
4	X→				
5	1				
6	2				
7	PNT				
8	PNT				
9	2				

STEP	KEY	CODE	x	y	z
0	↑		2	2	
1	↑		2	2	2
2	STOP		O or K	Wt	Diam
3	R↑		D	O or K	Wt
4	PNT		D		
5	R↑		Wt	D	O or K
6	X→				
7	3				
8	5				
9	PNT		Wt		
0	1				
1	2		12	D, in	
2	÷			D, ft	
3	y→				
4	1				
5	1				
6	↓			O or K	O or K
7	0		0	O or K	
8	X<Y				
9	0				
0	1				
1	8				
2	3				
3	X( )				
4	3		Z_B		
5	↑			Z_B	
6	X( )				
7	4		Z_S	Z_B	
8	−			$Z_B - Z_S$	
9	1				
0	0				
1	0		100	$Z_B - Z_S$	
2	÷			K	
018 3	↓		K		
4	PNT				
5	PNT				
6	X→				
7	6				
8	↑			K	
9	X( )				
0	3				
1	5		Wt		
2	↑			Wt	K
3	1				
4	E EXP				
5	3		1000	Wt	K
6	÷			Wt/ft	K
7	↓		Wt/ft	K	K
8	X			W·K	K
9	y→				

STEP	KEY	CODE	x	y	z
0200 0	9				
1	X( )				
2	1				
3	1		D, ft		K
4	R↑		K	D	
5	X			A	
6	y→				
7	0				
8	1				
9	0				
0210 0	3		3		
1	↑		3	3	
2	↑		3	3	3
3	STOP		θ	F_T	
4	X⇄Y		F_T	θ	
5	PNT		F_T		
6	X→				
7	1				
8	X→				
9	3				
0	4				
1	X⇄Y	•	θ	F_T	
2	PNT				
3	PNT				
4	X→				
5	2				
6	X( )				
7	0		R		
8	↑			R	
9	0		0	R	
0	X<Y				
1	0				
2	3				
3	2				
4	4				
5	FMT				
6	FMT				
7	W				
8	1				
9	N				
0	D				
1	S				
2	FMT				
0243	4		4		
4	↑		4	4	
5	↑		4	4	4
6	STOP		AZ	WIND	Z
7	R↑		Z	AZ	W
8	PNT		Z		
9	R↑		W	Z	AZ

STEP	KEY	CODE	x	y	z
0	PNT		W		
1	R↑		AZ	W	Z
2	PNT				
3	PNT				
4	y→				
5	3				
6	5				
7	X→				
8	3				
9	6				
0	b		NW		Z
1	X⇄Y			NW	Z
2	3		3		
3	X			3NW	
4	3				
5	6		36		
6	+			3NW+36	Z
7	y→				
8	a				
9	R↑		Z		
0	X→				
1	IND				
2	a				
3	X( )				
4	3				
5	5		W		
6	X⇄Y			W	
7	1				
8	•				
9	6				
0	8				
1	7				
2	8		1.6878	W, KNOTS	
3	X			V, fps	
4	1		1		
5	X→				
6	+				
7	a				
8	y→				
9	IND				
0	a				
1	X→				
2	+				
3	a				
4	X( )				
5	3				
6	6		AZ		
7	X→				
8	IND				
9	a				

STEP	KEY	CODE	x	y	z	STEP	KEY	CODE	x	y	z
030 0	2		2			0	↑			SIN θ	
1	X→					1	X( )				
2	−					2	6		K	SIN θ	
3	a					3	×			J	
4	X( )					4	X( )				
5	IND					5	5		Z	J	
6	a		Z			6	X⇄Y		J	Z	
7	↑			Z		7	−			Z'	
8	X( )					8	↑			J	Z'
9	4		Z_s	Z		9	X( )				
0	X=Y					0	4		Z_s		
1	0					1	R↑		Z'	Z_s	J
2	3					2	X>Y				
3	2					3	0				
4	4					4	3				
5	1		1			5	9				
6	X→					6	3				
7	+					7	X( )				
8	b					8	7		K RED	Z_s	J
9	GoTo					9	X⇄Y			K RED	
0	0					0	1				
1	2					1	0		10	K RED	
2	4					2	X=Y				
3	3					3	1				
032 4	1		NW=1			4	2				
5	X→					5	6				
6	b					6	7				
7	4					7	X→				
8	1		41			8	7				
9	↑			41		9	X→				
0	X( )					0	÷				
1	0		r	41		1	6				
2	+			r+41		2	X→				
3	y→					3	÷				
4	a					4	9				
5	X( )					5	X→				
6	IND					6	÷				
7	a		θ_B			7	1				
8	X→					8	0				
9	2					9	GoTo				
0	7					0	3				
1	X→					1	4				
2	3					2	7				
3	0					039 3	X→		Z'	Z_s	J
034 4	GoTo		} PLOT ROUTINE			4	5				
5	LABEL					5	PNT		Z'		
6	÷					6	R↑		J	Z'	Z_s
034 7	X( )					7	PNT		J		
8	2		θ			8	X→				
9	SIN X		SIN θ			9	+				

STEP	KEY	CODE	x	y	z
040 0	2				
1	3				
2	X( )				
3	2		θ		
4	COS X		cos θ		
5	↑			cos θ	
6	X( )				
7	6		K	cosθ	
8	X→				
9	+				
0	2				
1	4				
2	X			Kcosθ	
3	X( )				
4	2				
5	8		Σβ		
6	cos X		cos Σβ	Kcosθ	
7	X⇌Y		Kcosθ	cos Σβ	
8	X			h	
9	X⇌Y		h	Kcosθ	
0	PNT		h		
1	X→				
2	+				
3	2				
4	1				
5	X( )				
6	2				
7	8		Σβ		
8	SIN X		SIN Σβ	Kcosθ	
9	X			i	
0	↓		i		
1	PNT		i		
2	X→				
3	+				
4	2				
5	2				
6	X( )				
7	2				
8	3		J		
9	PNT		J		
0	X( )				
1	2				
2	1		H		
3	PNT		H		
4	X( )				
5	2				
6	2		I		
7	PNT		I		
8	PNT				
9	X( )				

STEP	KEY	CODE	x	y	z
0	2				
1	4		ΣK		
2	PNT		ΣK		
3	X( )				
4	9		W		
5	X→				
6	+				
7	2				
8	5				
9	X( )				
0	2				
1	5		ΣW		
2	PNT		ΣW		
046 3	b		NW		
4	↑			n*	
5	1		1	n	
6	+			$n'=n+1$	
7	3		3	n'	
8	X			$3n'$	
9	3				
0	6		36	$3n'$	
1	+			$3n'+36$	
2	X( )				
3	0		r	$3n'+36$	
4	+			$3n'+36+r$	
5	y→				
6	a				
7	X( )				
8	IND				
9	a		Z_{n+1}		
0	↑			Z_{n+1}	
1	X( )				
2	5		Z	Z_{n+1}	
3	X>Y				
4	0				
5	4				
6	9				
7	7				
8	1		1		
9	X→				
0	+				
1	b				
2	GoTo				
3	0				
4	4				
5	6				
6	3				
049 7	3		3		
8	X→				
9	−				

*NOTE: For CLARITY n = NW in STEP No 0464 — 0629

STEP	KEY	CODE	x	y	z
050 0	a				
1	X( )				
2	IND				
3	a		Z_n	Z_{n+1}	
4	X→				
5	3				
6	5				
7	X⇄Y		Z_{n+1}	Z_n	
8	−			ΔZ_W	
9	1		1		
0	X→				
1	+				
2	a				
3	X( )				
4	IND				
5	a		V_n	ΔZ_W	
6	↑			V_n	ΔZ_W
7	3		3		
8	X→				
9	+				
0	a				
1	X( )				
2	IND				
3	a		V_{n+1}	V_n	ΔZ_W
4	X⇄Y		V_n	V_{n+1}	
5	−		V_n	ΔV	
6	X⇄Y		ΔV	V_n	ΔZ_W
7	R↑		ΔZ_W	ΔV	V_n
8	÷			dV/dZ	
9	X→				
0	3				
1	6				
2	X( )				
3	3				
4	5		Z_n	dV/dZ	V_n
5	R↑		V_n	Z_n	dV/dZ
6	X→				
7	3				
8	5				
9	X( )				
0	5		Z	Z_n	
1	−			ΔZ_0	dV/dZ
2	y→				
3	3				
4	7				
5	↓		ΔZ_0	dV/dZ	
6	X			ΔV_0	
7	X( )				
8	3				
9	5		V_n	ΔV_0	

STEP	KEY	CODE	x	y	z
0	+			V	
1	y→				
2	0				
3	1				
4	3				
5	1				
6	•				
7	6				
8	8				
9	7				
0	8		1.6878	V_{fps}	
1	÷			V_{KNOTS}	
2	↓		V_{KNOTS}		
3	PNT		V_{KNOTS}		
4	↑			V	
5	0		0	V	
6	X=Y				
7	1				
8	0				
9	1				
0	8				
1	1		1		
2	X→				
3	+				
4	a				
5	X( )				
6	IND				
7	a		AZ_{n+1}		
8	↑			AZ_{n+1}	
9	3		3		
0	X→				
1	−				
2	a				
3	X( )				
4	IND				
5	a		AZ_n	AZ_{n+1}	
6	−			ΔAZ_t	
7	1				
8	8				
9	0		180	ΔAZ_t	
0	X<Y				
1	0				
2	6				
3	0				
4	6				
5	CHG S		−180	ΔAZ_t	
6	X>Y				
7	0				
8	6				
9	1				

STEP	KEY	CODE	x	y	z	STEP	KEY	CODE	x	y	z
060 0	5					0	0				
1	GoTo					1	6				
2	0					2	5				
3	6					3	8				
4	1					065 4	3				
5	9					5	6				
060 6	3					6	0		360	AZ_t	
7	6					7	—			AZ	
8	0		360	ΔAZ_t		065 8	y→				
9	—			ΔAZ		9	1				
0	GoTo					0	4				
1	0					1	X( )				
2	6					2	2				
3	1					3	7		$\Sigma\beta+\beta_B$	AZ	
4	9					4	X⇌y		AZ	$\Sigma\beta+\beta_B$	
061 5	3					5	PNT		AZ		
6	6					6	X⇌y		$\Sigma\beta+\beta_B$	AZ	
7	0		360			7	PNT		$\Sigma\beta+\beta_B$	AZ	
8	+			ΔAZ		8	—			α_t	
61 9	X( )					9	0		0	α_t	
0	3					0	X>Y				
1	6		ΔZ_W	ΔAZ		1	0				
2	÷			dAZ/dZ		2	6				
3	X( )					3	8				
4	3					4	0				
5	7		ΔZ_0	dAZ/dZ		5	GoTo				
6	×			ΔAZ_0		6	0				
7	X( )					7	6				
8	IND					8	8				
9	a		AZ_n	ΔAZ_0		9	4				
0	+			AZ_t		068 0	3				
1	3					1	6				
2	6					2	0		360	α_t	
3	0		360	AZ_t		3	+			α	
4	X<Y					068 4	y→				
5	0					5	1				
6	6					6	5				
7	5					7	↓		α		
8	4					8	PNT		α		
9	0		0	AZ_t		9	cos X		$\cos\alpha$		
0	X<Y					0	X^2		$\cos^2\alpha$		
1	0					1	↑			$\cos^2\alpha$	
2	6					2	X( )				
3	5					3	2		θ		
4	8					4	cos X		$\cos\theta$		
5	3					5	X^2		$\cos^2\theta$	$\cos^2\alpha$	
6	6					6	×			$\cos^2\cos^2$	
7	0		360	AZ_t		7	1		1		
8	+			AZ		8	X⇌y		$\cos^2\cos^2$	1	
9	GoTo					9	—			$\cos^2\phi$	

STEP	KEY	CODE	x	y	z
0700 0	↓		$\cos^2\phi$		
1	$\sqrt{x}$		$\cos\phi$		
2	X→				
3	2				
4	9				
5	↑			$\cos\phi$	
6	X( )				
7	1				
8	3		V	$\cos\phi$	
9	X			V_N	
0	y→				
1	1				
2	6				
3	X( )				
4	5		Z		
5	↑		Z	Z	
6	↑		Z	Z	Z
7	3				
8	6				
9	5				
0	0				
1	0		36500	Z	
2	X>Y				
3	0				
4	7				
5	4				
6	1				
7	•				
8	9				
9	5				
0	5				
1	2				
2	8				
3	ENT E				
4	5				
5	CHG S		μ	Z	Z
6	GoTo				
7	0				
8	7				
9	6				
0	4				
074 1	6				
2	•				
3	8				
4	4				
5	1				
6	6				
7	4				
8	ENT E				
9	1				

STEP	KEY	CODE	x	y	z
0	1				
1	CHG S		6.8^{-11}	Z	Z
2	X			$6.8^{-11}Z$	Z
3	1				
4	•				
5	2				
6	0				
7	5				
8	ENT E				
9	5				
0	CHG S		1.2^{-5}	$6.8^{-11}Z$	Z
1	X⇄Y		$6.8^{-11}Z$	1.2^{-5}	
2	−			μ	Z
3	↓		μ	Z	Z
076 4	X→				
5	3				
6	3				
7	X( )				
8	7				
9	5		a_1	Z	Z
0	X			a_1Z	
1	X( )				
2	7				
3	6		a_0	a_1Z	Z
4	+			a_0+a_1Z	Z
5	↓		a_0+a_1Z	Z	Z
6	X			$\ln \rho/\rho_0$	
7	↓		$\ln \rho/\rho_0$	Z	
8	e^x		ρ/ρ_0		
9	↑			ρ/ρ_0	
0	•				
1	0				
2	0				
3	2				
4	3				
5	7				
6	8		ρ_0	ρ/ρ_0	
7	X			ρ (slugs/ft^3)	
8	y→				
9	0				
0	3				
1	2				
2	3				
3	2				
4	•				
5	1				
6	7				
7	4		32.174	ρ	
8	X			ρ (lbs/ft^3)	
9	X( )				

STEP	KEY	CODE	x	y	z
080 0	1				
1	6		V_N	ρ	
2	X			ρV_N	
3	X( )				
4	1				
5	1		D	ρV_N	
6	X			$\rho V_N D$	
7	X( )				
8	3				
9	3		μ	$\rho V_N D$	
0	÷			R	
1	↓		R		
2	PNT		R		
3	↑			R	
4	X( )				
5	1				
6	2		π OR C_D	R	
7	↑			π OR C_D	R
8	π		π	π OR C_D	R
9	X=Y				
0	0				
1	8				
2	2				
3	9				
4	GoTo				
5	0				
6	9				
7	9				
8	3				
082 9	1		1	π	R
0	R↑		R	1	π
1	X>Y				
2	0				
3	8				
4	5				
5	8				
6	↑			R	1
7	1				
8	0				
9	.				
0	9		10.9	R	1
1	X⇄Y		R	10.9	1
2	÷		R	10.9/R	
3	TAB				
4	4		LOG R		
5	↑			LOG R	10.9/R
6	.				
7	8				
8	7		.87	LOG R	10.9/R
9	X⇄Y		LOG R	.87	

STEP	KEY	CODE	x	y	z
0	−			.87−LOG R	10.9/R
1	↓		.87−LOG R	10.9/R	
2	÷			C_D	
3	GoTo				
4	0				
5	9				
6	9				
7	3				
085 8	↑			R	1
9	9		9	R	1
0	X<Y				
1	0				
2	8				
3	7				
4	1				
5	1		$1 = n_{CD}$	R	1
6	GoTo				
7	0				
8	9				
9	5				
0	8				
087 1	9				
2	0				
3	0		900	R	
4	X<Y				
5	0				
6	8				
7	8				
8	5				
9	2		$2 = n_{CD}$	R	
0	GoTo				
1	0				
2	9				
3	5				
4	8				
088 5	4				
6	5				
7	0				
8	0		4500	R	
9	X<Y				
0	0				
1	9				
2	0				
3	0				
4	3		$3 = n_{CD}$	R	
5	GoTo				
6	0				
7	9				
8	5				
9	8				

STEP	KEY	CODE	x	y	z	STEP	KEY	CODE	x	y	z
090 0	9					0	7				
1	ENTE					1	7		$7 = n_{cp}$	R	
2	3		9000	R		2	GoTo				
3	X<Y					3	0				
4	0					4	9				
5	9					5	5				
6	1					6	8				
7	4					095 7	8		$8 = n_{cp}$	R	
8	4		$4 = n_{cp}$	R		095 8	↑			n_{cp}	R
9	GoTo					9	3		3	n_{cp}	
0	0					0	×			$3n_{cp}$	R
1	9					1	7				
2	5					2	4		74	$3n_{cp}$	R
3	8					3	+			$3n_{cp}+74$	
091 4	4					4	y→				
5	ENTE					5	a				
6	4		40 000	R		6	X( )				
7	X<Y					7	IND				
8	0					8	a		R_B		R
9	9					9	TAB				
0	2					0	4		LOG R_B		R
1	8					1	X⊃4			LOG R_B	R
2	5		$5 = n_{cp}$	R		2	2		2		
3	GoTo					3	X→				
4	0					4	+				
5	9					5	a				
6	5					6	R↑		R		LOG R_B
7	8					7	TAB				
092 8	5					8	4		LOG R		LOG R_B
9	ENTE					9	R↑		LOG R_B	LOG R	
0	4		50000	R		0	−			$LR-LR_B$	
1	X<Y					1	X( )				
2	0					2	IND				
3	9					3	a		K_R	$LR-LR_B$	
4	4					4	×			$K_R(\ )$	
5	(2)					5	1		1		
6	6		$6 = n_{cp}$	R		6	X→				
7	GoTo					7	−				
8	0					8	a				
9	9					9	X( )				
0	5					0	IND				
1	8					1	a		C_{DA}	$K_R(\ )$	
094 2	2					2	+			C_D	
3	5					099 3	↓		C_D		
4	ENTE					4	PNT		C_D		
5	4		250000	R		5	↑			C_D	
6	X<Y					6	X( )				
7	0					7	1				
8	9					8	6		V_N	C_D	
9	5					9	X²		V_N^2	C_D	

STEP	KEY	CODE	X	Y	Z	STEP	KEY	CODE	X	Y	Z
100 0	↑			V_N^2	C_D	0	cos X		cos α	$D_T\cos\theta$	
1	X( )					1	X			D_W	
2	3					2	X⇄Y		D_W	cos α	
3	2		ρ (slug)	V_N^2		3	X→				
4	X			ρV_N^2		4	1				
5	2		2	ρV_N^2		5	7				
6	÷			q	C_D	6	X→				
7	↓		q	C_D		7	+				
8	X→					8	2				
9	3					9	6				
0	2					0	PNT		D_W		
1	PNT		q			1	X( )				
2	X			qC_D		2	2				
3	X( )					3	9		cos ϕ	cos α	
4	1					4	X		cos ϕ	cos ϕ cos α	
5	0		A	qC_D		5	↑			cos ϕ	cos ϕ cos α
6	X			D_T		6	X( )				
7	↓		D_T			7	3				
101 8	X→					8	1		D_T	cos ϕ	cos ϕ cos α
9	3					9	R↑		cos ϕ cos α	D_T	cos ϕ
0	1					0	X			D_H	
1	PNT		D_T			1	↓		D_H	cos ϕ	
2	↑			D_T		2	PNT		D_H		
3	X( )					3	X→				
4	2		θ			4	1				
5	↑			θ	D_T	5	8				
6	9					6	X( )				
7	0		90	θ		7	3				
8	X>Y					8	1		D_T	cos ϕ	
9	1					9	X			$D_T\cos\phi$	
0	0					0	X( )				
1	4					1	1				
2	4					2	5		α		
3	0		0	θ	D_T	3	SIN X		SIN α	$D_T\cos\phi$	
4	R↑		D_T	0	θ	4	X			D_S	
5	X⇄Y		0	D_T	θ	5	↓		D_S		
6	X=Y					6	PNT		D_S		
7	1					7	PNT				
8	2					8	CNT				
9	0					9	CNT				
0	4					0	CNT				
1	↓		D_T	θ		1	↑			D_S	
2	X⇄Y		θ	D_T		2	X( )				
3	↑			θ	D_T	3	2		θ	D_S	
104 4	↓		θ	D_T		4	cos X		cos θ	D_S	
5	cos X		cos θ	D_T		5	↑			cos θ	D_S
6	X			$D_T\cos\theta$		6	X( )				
7	X( )					7	1		F_T OR T	cos θ	D_S
8	1					8	X			T cos θ	
9	5		α	$D_T\cos\theta$		9	X( )				

STEP	KEY	CODE	X	Y	Z	STEP	KEY	CODE	X	Y	Z
110 0	1					0	2				
1	8		D_H	$T\cos\theta$	D_S	1	cos X		$\cos\theta_1$	$\cos\beta_1$	
2	+			$T\cos\theta+D_H$	D_S	2	X			$\cos\theta_1\cos\beta_1$	
3	↓		$T\cos\theta+D_H$	D_S		3	X( )				
4	X→					4	2				
5	2					5	0		$T\cos+D_H$	$\cos\theta_1\cos\beta_1$	
6	0					6	X⇄Y		$\cos\theta_1\cos\beta_1$	$T\cos+D_H$	
7	÷			$\tan\beta_1$		7	÷			T_1	
8	↓		$\tan\beta_1$			8	↓		T_1		
9	arc					9	X→				
0	TAN X		β_1			0	1				
1	GoTo					1	PNT				
2	LABEL					2	PNT				
3	π					3	X( )				
111 4	PNT		β_1			4	2				
5	X→					5	7		$\Sigma\beta+\beta_n$		
6	+					6	↑			$\Sigma\beta+\beta_n$	
7	2					7	3				
8	7					8	6				
9	X→					9	0		360	$\Sigma\beta+\beta_n$	
0	+					0	X<Y				
1	2					1	1				
2	8					2	1				
3	cos X		$\cos\beta_1$			3	8				
4	↑			$\cos\beta_1$		4	6				
5	X( )					5	0		0	$\Sigma\beta+\beta_n$	
6	2		θ			6	X>Y				
7	SIN X		$\sin\theta$	$\cos\beta_1$		7	1				
8	↑			$\sin\theta$	$\cos\beta_1$	8	1				
9	X( )					9	9				
0	1		F_T OR T	$\sin\theta$		0	2				
1	X			$T\sin\theta$		1	GoTo				
2	X( )					2	1				
3	9		W			3	1				
4	−			$T_{\sin}-W$		4	9				
5	X( )					5	6				
6	1					118 6	−		360	$\Sigma\beta+\beta_n$	
7	7		D_W	$T_{\sin}-W$		7	GoTo				
8	−			$T_{\sin}-W-D_W$		8	1				
9	X( )					9	1				
0	2					0	9				
1	0		$T\cos\theta+D_H$			1	6				
2	÷			( )/( )	$\cos\beta_1$	119 2	3				
3	↓		( )/( )	$\cos\beta_1$		3	6				
4	X			$\tan\theta_1$		4	0		360	$\Sigma\beta+\beta_n$	
5	↓		$\tan\theta_1$	$\cos\beta_1$		5	+			$\Sigma\beta+\beta_n$	
6	arc					119 6	y→				
7	TAN X		θ_1			7	2				
8	PNT		θ_1			8	7				
9	X→					9	GoTo				

STEP	KEY	CODE	X	Y	Z	STEP	KEY	CODE	X	Y	Z
1200 0	1					0	FMT				
1	2					1	FMT				
2	1					2	C				
3	8					3	A				
1204	X( )					4	B				
5	2		$\theta_1=90$			5	L				
6	PNT					6	E				
7	X( )					7	CNT				
8	1		FLOAT			8	H				
9	↑			T		9	O				
0	X( )					0	R				
1	9		W	T		1	FMT				
2	−			T_1		2	GoTo				
3	y→					3	1				
4	/					4	2				
5	↓		T_1			5	7				
6	PNT		T_1			6	7				
7	PNT					1267	FMT				
121 8	X( )					8	FMT				
9	1		T_1			9	O				
0	↑			T_1		0	N				
1	0		0	T_1		1	CNT				
2	X<Y					2	S				
3	1					3	U				
4	2					4	R				
5	4					5	F				
6	1					6	FMT				
7	FMT					1277	FMT				
8	FMT					8	FMT				
9	T					9	A				
0	E					0	L				
1	N					1	T				
2	6					2	FMT				
3	•					3	X( )				
4	0					4	5		Z		
5	FMT					5	PNT		Z		
6	GoTo					6	FMT				
7	1					7	FMT				
8	2					8	H				
9	7					9	T				
0	7					0	FMT				
1241	X( )					1	X( )				
2	2		θ_1			2	2				
3	↑			θ_1		3	3		J		
4	0		0	θ_1		4	PNT		J		
5	X<Y					5	FMT				
6	0					6	FMT				
7	3					7	T				
8	4					8	E				
9	4					9	N				

STEP	KEY	CODE	X	Y	Z	STEP	KEY	CODE	X	Y	Z
130 0	S					0	2				
1	FMT					1	6		ΣD_W		
2	X( )					2	PNT		ΣD_W		
3	1		T			3	FMT				
4	PNT		T			4	FMT				
5	FMT					5	H				
6	FMT					6	,				
7	C					7	I				
8	•					8	,				
9	E					9	L				
0	L					0	FMT				
1	E					1	X( )				
2	V					2	2				
3	•					3	1		H		
4	A					4	PNT		H		
5	N					5	↑			H	
6	G					6	X( )				
7	FMT					7	2				
8	X( )					8	2		I	H	
9	2		θ			9	PNT		I		
0	PNT		θ			0	To Pol		L	δ	
1	FMT					1	X→				
2	FMT					2	3				
3	L					3	5				
4	E					4	PNT		L		
5	N					5	X( )				
6	G					6	3				
7	T					7	0		β_B	δ	
8	H					8	↑			β_B	δ
9	FMT					9	9				
0	X( )					0	0		90	β_B	δ
1	2					1	−			β_B-90	
2	4		ΣK			2	↓		β_B-90	δ	
3	PNT		ΣK			3	X⇄Y		δ	β_B-90	
4	FMT					4	−			AZ_B	
5	FMT					5	3				
6	W					6	6				
7	T					7	0		360	AZ_B	
8	FMT					8	X<Y				
9	X( )					9	1				
0	2					0	4				
1	5		ΣW			1	0				
2	PNT		ΣW			2	4				
3	FMT					3	0		0	AZ_B	
4	FMT					4	X>Y				
5	V					5	1				
6	•					6	4				
7	D					7	1				
8	FMT					8	0				
9	X( )					0	GoTo				

STEP	KEY	CODE	x	y	z
140 0	1				
1	4				
2	1				
3	4				
140 4	−		360	AZB'	
5	GoTo				
6	1				
7	4				
8	1				
9	4				
141 0	3				
1	6				
2	0		360	AZB	
3	+		360	AZB'	
141 4	↓		AZB		
5	FMT				
6	FMT				
7	A				
8	Z				
9	•				
0	T				
1	O				
2	CNT				
3	B				
4	L				
5	N				
6	FMT				
7	PNT		AZB		
8	X→				
9	3				
0	6				
1	X( )				
2	2				
3	3		J		
4	↑			J	
5	X( )				
6	3				
7	5		L	J	
8	÷		L	TAN E	
9	X⇄Y		TAN E	L	
0	ARC				
1	TAN X		E	L	
2	FMT				
3	FMT				
4	B				
5	•				
6	E				
7	L				
8	•				
9	A				

STEP	KEY	CODE	x	y	z
0	N				
1	G				
2	FMT				
3	PNT		E		
4	COS X		COS E	L	
5	÷			S.R	
6	↓		S.R		
7	FMT				
8	FMT				
9	S				
0	•				
1	R				
2	FMT				
3	PNT		S.R.		
4	X( )				
5	2				
6	7		ΣB+BB		
7	↑			ΣB+BB	
8	1				
9	8				
0	0		180	ΣB+BB	
1	−			AZC	
2	0		0	AZC	
3	X>Y				
4	1				
5	4				
6	8				
7	3				
8	GoTo				
9	1				
0	4				
1	8				
2	7				
148 3	3				
4	6				
5	0		360	AZC	
6	+		360	AZC'	
148 7	3				
8	6				
9	0		360	AZC	
0	X>Y				
1	1				
2	4				
3	9				
4	6				
5	−		360	AZC'	
149 6	↓		AZC		
7	FMT				
8	FMT				
9	C				

STEP	KEY	CODE	x	y	z
150 0	•				
1	A				
2	Z				
3	FMT				
4	PNT		AZ_C		
5	X( )				
6	3				
7	6		AZ_B		
8	↑			AZ_B	
9	9				
0	0		90	AZ_B	
1	X>Y				
2	1				
3	5				
4	1				
5	9				
6	4				
7	5				
8	0		450	AZ_B	
151 9	X⇄Y		AZ_B	450 a 90	
0	−			γ	
1	↓		γ		
2	↑		γ	γ	
3	SIN X		SIN γ	γ	
4	X⇄Y		γ	SIN γ	
5	COS X		COS γ	SIN γ	
6	↑			COS γ	SIN γ
7	X( )				
8	3				
9	5		L	COS γ	SIN γ
0	X		L	X	SIN γ
1	R↑		SIN γ	L	X
2	X			Y	X
3	R↑		X		Y
4	FMT				
5	FMT				
6	X				
7	CNT				
8	+				
9	E				
0	FMT				
1	PNT		X		
2	R↑		Y	X	
3	FMT				
4	FMT				
5	Y				
6	CNT				
7	+				
8	N				
9	FMT				

STEP	KEY	CODE	x	y	z
0	PNT		Y		
1	X( )				
2	0		r		
3	↑			r	
4	0		0	r	
5	X<Y				
6	1				
7	6				
8	0				
9	5				
156 0	FMT				
1	FMT				
2	O				
3	P				
4	T				
5	•				
6	E				
7	N				
8	T				
9	CLR				
0	O				
1	−				
2	N				
3	E				
4	W				
5	CNT				
6	P				
7	R				
8	O				
9	B				
0	CLR				
1	2				
2	−				
3	L				
4	O				
5	W				
6	CNT				
7	C				
8	Y				
9	C				
0	L				
1	E				
2	S				
3	FMT				
4	STOP		ON		
5	X→				
6	8				
7	↑			ON	r
8	0		0	ON	r
9	X=Y				

STEP	KEY	CODE	X	Y	Z	STEP	KEY	CODE	X	Y	Z
160 0	1					0	0				
1	6					1	0				
2	6					2	5				
3	0					3	4				
4	↑		0	0=r	Z=ON	4	↓		29	0	
160 5	3		3	r		5	GoTo				
6	+			r'		6	1				
7	y→					7	6				
8	0					8	3				
9	X⟨ ⟩					9	8				
0	7		K RED			166 0	0		0	0	
1	↑			K RED		1	X→				
2	1					2	a				
3	0		10	K RED		166 3	X→				
4	X>Y					4	IND				
5	1					5	a				
6	6					6	1		1		
7	3					7	X→				
8	0					8	+				
9	X→					9	a				
0	X					0	7				
1	6					1	5		75		
2	X→					2	↑			75	
3	X					3	a		RegNo	75	
4	9					4	X=Y				
5	X→					5	0				
6	X					6	0				
7	0					7	0				
8	1					8	0				
9	0					9	0		0		
163 0	0		0			0	GoTo				
1	X→					1	1				
2	7					2	6				
3	↑			0		3	6				
4	2					4	3				
5	1		21	0		5	LABEL				
6	X→					6	π				
7	a					7	↑			β_1	
163 8	y→					8	X⟨ ⟩				
9	IND					9	2		θ		
0	a					0	↑			θ	β_1
1	1		1	0		1	7				
2	X→					2	5		75	θ	β_1
3	+					3	X>Y				
4	a					4	1				
5	2					5	7				
6	9		29	0		6	1				
7	↑			29	0	7	4				
8	a		RegNo	29	0	8	X⟨ ⟩				
9	X=Y					9	2				

STEP	KEY	CODE	X	Y	Z	STEP	KEY	CODE	X	Y	Z
170 0	Θ		Yini+Δy	Θ	A_1	0	3				
1	X⇄Y			Yini+Δy	A_1	1	8				
2	Θ		Θ	Yini+Δy	A_1	2	X⇄Y		.3( )	6/5( )	
3	X<Y					3	CHG S		−.3( )	6/5( )	
4	1					4	FMT				
5	7					5	1				
6	1					6	2				
7	4					7	X( )				
8	1					8	3		Z_B		
9	8					9	↑			Z_B	
0	0		180		B_1	0	X( )				
1	R↑		B_1	180		1	5		Z	Z_B	
2	+			A_1		2	X<Y				
3	↑				B_1	3	1				
171 4	R↑		A_1			4	8				
5	GO TO					5	0				
6	1					6	6				
7	1					7	0		0		
8	1					8	↑		0	0	
9	4					9	FMT				
0	LABEL					0	1				
1	÷					1	↑				
2	X( )					2	5		5		
3	3		Z_B			3	↑		5	5	
4	↑			Z_B		4	↑		5	5	5
5	X( )					5	STOP		P	M	N
6	4		Z_S	Z_B		6	X→				
7	−			Z_B-Z_S		7	1				
8	0		0	Z_B-Z_S		8	9				
9	y→					9	y→				
0	3					0	3				
1	7					1	6				
2	FMT					2	R↑		N	P	M
3	1					3	X→				
4	3					4	3				
5	X( )					5	5				
6	3					6	FMT				
7	7		Z_B-Z_S			7	1				
8	↑			Z_B-Z_S		8	6				
9	↑			Z_B-Z_S	Z_B-Z_S	9	0		0		
0	•					0	↑		0	0	
1	3		.3			1	FMT				
2	×			.3( )		2	1				
3	6		6	.3( )	Z_B-Z_S	3	↑				
4	R↑		Z_B-Z_S	6	.3( )	4	X( )				
5	×			6( )		5	3				
6	5		5	6( )		6	5		N		
7	÷			6/5( )	.3( )	7	FMT				
8	↓		6/5( )	.3( )		8	1				
9	y→					9	5				

STEP	KEY	CODE	X	Y	Z
180 0	•				
1	0				
2	5		.05		
3	FMT				
4	1				
5	1				
180 6	X( )				
7	5		Z		
8	↑			Z	
9	X( )				
0	4		Z_s	Z	
1	−			$Z-Z_s$	
2	y→				
3	a				
4	X( )				
5	2				
6	1		H	$Z-Z_s$	
7	FMT				
8	1				
9	↑				
0	FMT				
1	1				
2	4				
3	a		$Z-Z_s$		
4	^			$Z-Z_s$	
5	X( )				
6	2				
7	2		I	$Z-Z_s$	
8	FMT				
9	1				
0	↑				
1	FMT				
2	↓				
3	1				
4	2				
5	0		120		
6	↑			120	
7	3				
8	0		30	120	
9	CHG S		−30	120	
0	FMT				
1	1				
2	2				
3	a		$Z-Z_s$		
4	↑			$Z-Z_s$	
5	X( )				
6	2		θ	$Z-Z_s$	
7	FMT				
8	1				
9	↑				

STEP	KEY	CODE	X	Y	Z
0	FMT				
1	1				
2	FMT				
3	0				
4	FMT				
5	CNT				
6	X( )				
7	3				
8	4		F_T		
9	↑			F_T	
0	X( )				
1	0				
2	1				
3	9		P	F_T	
4	TAB				
5	9		F_T Rnd	F_T	
6	↑			F_TR	
7	↑			F_TR	F_TR
8	2		2	F_TR	
9	÷			$F_TR/2$	
0	R↑		F_TR	2	$F_TR/2$
1	×			$2F_TR$	
2	↓		$2F_TR$	$F_TR/2$	
3	X⇌Y		$F_TR/2$	$2F_TR$	
4	CHG S		$-F_TR/2$	$2F_TR$	
5	FMT				
6	1				
7	2				
8	X( )				
9	3		Z_B		
0	↑			Z_B	
1	X( )				
2	5		Z	Z_B	
3	X<Y				
4	1				
5	9				
6	0				
7	2				
8	X( )				
9	3				
0	7		Z_B-Z_s		
1	↑			Z_B-Z_s	
2	0		0	Z_B-Z_s	
3	FMT				
4	1				
5	↑				
6	X( )				
7	3				
8	6		M		
9	FMT				

190	0	:				
	1	5				
190	2	a		$Z-Z_s$		
	3	↑			$Z-Z_s$	
	4	X( )				
	5	0				
	6	0				
	7	1		T	$Z-Z_s$	
	8	FMT				
	9	1				
	0	↑				
	1	FMT				
	2	1				
	3	FMT				
	4	T				
	5	FMT				
	6	CNT				
	7	2				
	8	4		24		
	9	↑			24	
	0	6		6	24	
	1	CHGS		−6	24	
	2	FMT				
	3	/				
	4	2				
	5	a		$Z-Z_s$		
	6	↑			$Z-Z_s$	
	7	X( )				
	8	3				
	9	2		9	$Z-Z_s$	
	0	FMT				
	1	/				
	2	↑				
	3	FMT				
	4	/				
	5	FMT				
	6	X				
	7	FMT				
	8	CNT				
	9	X( )				
	0	3				
	1	8		$.3(Z_B-Z_s)$		
	2	X→				
	3	a				
	4	↑			.3( )	
	5	X( )				
	6	3				
	7	7		Z_B-Z_s		
	8	↑			Z_B-Z_s	.3( )
	9	•				

	0	7		.7	Z_B-Z_s	.3( )
	1	X			.7( )	
	2	↓		.7( )	.3( )	
	3	CHGS		−.7( )	.3( )	
	4	FMT				
	5	1				
	6	3				
	7	a		.3( )		
	8	↑			.3( )	
	9	3		3	.3( )	
	0	÷			.1( )	
	1	4		4	.1( )	
	2	X		4	.4( )	
	3	↑			4	.4( )
	4	↓		4	.4( )	.4( )
	5	÷			.1( )	.4( )
	6	/				
	7	/		11	.1( )	
	8	X			1.1( )	.4( )
	9	↓		1.1( )	.4( )	
	0	CHGS		−1.1( )	.4( )	
	1	FMT				
	2	1				
	3	2				
	4	X( )				
	5	3		Z_B		
	6	↑			Z_B	
	7	X( )				
	8	5		Z	Z_B	
	9	$X<Y$				
	0	2				
	1	0				
	2	0				
	3	3				
	4	0		0		
	5	↑		0	0	
	6	FMT				
	7	/				
	8	↑				
	9	X( )				
	0	3				
	1	5		N		
	2	FMT				
	3	/				
	4	5				
	5	0		0		
	6	↑		0	0	
	7	X( )				
	8	3				
	9	5		N	0	

2000	FMT				
1	1				
2	6				
2003	X( )				
4	2				
5	1		H		
6	CHG S		-H		
7	↑			-H	
8	X( )				
9	2				
0	2		I	-H	
1	CHG S		-I	-H	
2	FMT				
3	1				
4	↑				
5	FMT				
6	↓				
7	Go To				
8	0				
9	3				
0	4				
1	7				
2	END				

Storage Registers

STORAGE		
b	NW Wind code	
a	IND. ADRES.	
000	r cycle code	
001	$F_T \to T$	
002	Θ	
003	Z_B	
004	Z_S	
005	Z	
006	K	
007	K Red. 0/10	
008	ON option code	
009	W	
010	A	
011	D	
012	π or C_D	
013	V	
014	AZ	
015	α	
016	V_N	
017	D_W	
018	D_H	
019		
020	$T\cos\theta + D_H$	
021	H	SUMMARIES
022	I	SUMMARIES
023	J	SUMMARIES
024	ΣK	SUMMARIES
025	ΣW	SUMMARIES
026	ΣDW	SUMMARIES
027	$\Sigma \beta + \beta_B$	SUMMARIES
028	$\Sigma \beta$	SUMMARIES
029	$\cos\phi$	
030	β_A	
031	D_T	
032	$\rho \to q$	
033	μ	
034	E_T	
035	Temp	
036	Temp	
037	Temp	
038	Temp	
039	Z_1	WIND FIELD

040	V_1	WIND FIELD
041	AZ_1	WIND FIELD
042	Z_2	WIND FIELD
043	V_2	WIND FIELD
044	AZ_2	WIND FIELD
045	Z_3	WIND FIELD
046	V_3	WIND FIELD
047	AZ_3	WIND FIELD
048	Z_4	WIND FIELD
049	V_4	WIND FIELD
050	AZ_4	WIND FIELD
051	Z_5	WIND FIELD
052	V_5	WIND FIELD
053	AZ_5	WIND FIELD
054	Z_6	WIND FIELD
055	V_6	WIND FIELD
056	AZ_6	WIND FIELD
057	Z_7	WIND FIELD
058	V_7	WIND FIELD
059	AZ_7	WIND FIELD
060	Z_8	WIND FIELD
061	V_8	WIND FIELD
062	AZ_8	WIND FIELD
063	Z_9	WIND FIELD
064	V_9	WIND FIELD
065	AZ_9	WIND FIELD
066	Z_{10}	WIND FIELD
067	V_{10}	WIND FIELD
068	AZ_{10}	WIND FIELD
069	Z_{11}	WIND FIELD
070	V_{11}	WIND FIELD
071	AZ_{11}	WIND FIELD
072	Z_{12}	WIND FIELD
073	V_{12}	WIND FIELD
074	AZ_{12}	WIND FIELD
075	a_1	LOADED FROM DATA CARD
076	a_0	LOADED FROM DATA CARD
077	R_B	1
078	C_{DB}	1
079	K_R	1

080	R_B	2	LOADED FROM DATA CARD
081	C_{DB}	2	LOADED FROM DATA CARD
082	K_R	2	LOADED FROM DATA CARD
083	R_B	3	LOADED FROM DATA CARD
084	C_{DB}	3	LOADED FROM DATA CARD
085	K_R	3	LOADED FROM DATA CARD
086	R_B	4	LOADED FROM DATA CARD
087	C_{DB}	4	LOADED FROM DATA CARD
088	K_R	4	LOADED FROM DATA CARD
089	R_B	5	LOADED FROM DATA CARD
090	C_{DB}	5	LOADED FROM DATA CARD
091	K_R	5	LOADED FROM DATA CARD
092	R_B	6	LOADED FROM DATA CARD
093	C_{DB}	6	LOADED FROM DATA CARD
094	K_R	6	LOADED FROM DATA CARD
095	R_B	7	LOADED FROM DATA CARD
096	C_{DB}	7	LOADED FROM DATA CARD
097	K_R	7	LOADED FROM DATA CARD
098	R_B	8	LOADED FROM DATA CARD
099	C_{DB}	8	LOADED FROM DATA CARD
100	K_R	8	LOADED FROM DATA CARD
101	PLOT		USED IN PLOT ROUTINE
102	PLOT		USED IN PLOT ROUTINE
103	PLOT		USED IN PLOT ROUTINE
104	PLOT		USED IN PLOT ROUTINE
105	PLOT		USED IN PLOT ROUTINE
106	PLOT		USED IN PLOT ROUTINE
107	PLOT		USED IN PLOT ROUTINE
108	PLOT		USED IN PLOT ROUTINE

3.5.7 SAMPLE INPUT/OUTPUT PRINT AND PLOT

The following are copies of the HP Printed Tape for Test No. 5 in Section 4.

A. No Intermediate
Altitude Print

```
PROG#77.007P
3-DIM.TETHER
TEST
          5.000*
RUN#1
      10000.000
       4000.000
          3.142

          1.250
         20.000
        180.000

       3200.000
         85.000

WINDS
      10000.000
         25.000
        180.000

       8500.000
         40.000
        225.000

       7000.000
         50.000
        270.000

       6000.000
         60.000
        300.000

       4000.000
         20.000
        315.000
```

```
ON SURF
ALT
       4003.890
HT
       5996.110
TENS
       3271.459
C.ELEV.ANG
         35.932
LENGTH
       7290.000
WT
        145.800
V.D
       1122.263
H,I,L
       1476.213
       3000.984
       3344.415
AZ.TO BLN
         63.807
B.EL.ANG
         60.849
S.R
       6865.745
C.AZ
         85.548
X +E
       3000.984
Y +N
       1476.213
OPT.ENT
0-NEW PROB
2-LOW CYCLES
```

RUN#
2.000
B.ALT/AZ
8500.000
225.000

3200.000
5.000

CABLE HOR
ALT
8089.653
HT
410.347
TENS
3209.964
C.ELEV.ANG
-0.136
LENGTH
8280.000
WT
165.600
V.D
120.934
H,I,L
8267.066
2.584
8267.066
AZ.TO BLN
45.018
B.EL.ANG
2.842
S.R
8277.244
C.AZ
45.074
X +E
5847.525
Y +N
5843.871

RUN#
3.000
B.ALT/AZ
7000.000
270.000

3200.000
85.000

ON SURF
ALT
4007.570
HT
2992.430
TENS
3171.082
C.ELEV.ANG
56.523
LENGTH
3258.000
WT
65.160
V.D
477.637
H,I,L
1113.406
369.571
1173.139
AZ.TO BLN
108.362
B.EL.ANG
68.593
S.R
3214.170
C.AZ
113.851
X +E
1113.406
Y +N
-369.571

RUN#
4.000
B.ALT/AZ
6000.000
300.000

3200.000
85.000

ON SURF
ALT
4005.953
HT
1994.047
TENS
3174.239
C.ELEV.ANG
68.224
LENGTH
2070.000
WT
41.400
V.D
198.685
H,I,L
520.478
28.087
521.235
AZ.TO BLN
123.089
B.EL.ANG
75.351
S.R
2061.046
C.AZ
124.851
X +E
436.703
Y +N
-284.563
PROG#77.007P
3-DIM.TETHER
TEST

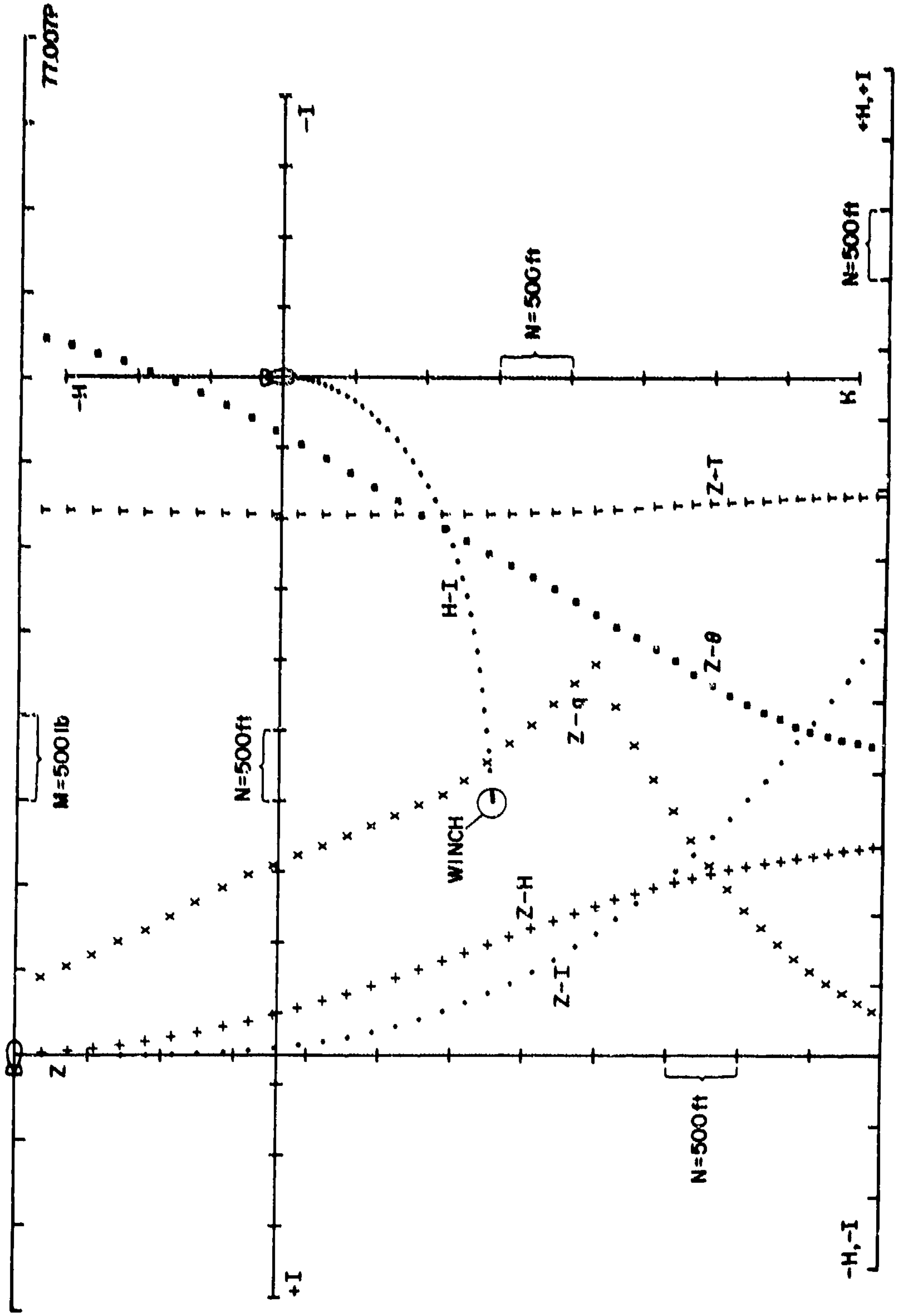
B. Full Print of
All Intermediate
Altitude Data

```
PROG#77.007P
3-DIM.TETHER
TEST
        5.000*
RUN#1
    10000.000
     4000.000
        3.142

        1.250
       20.000
      180.000

     3200.000
       65.000

WINDS
    10000.000
       25.000
      180.000

     8500.000
       40.000
      225.000

     7000.000
       50.000
      270.000

     6000.000
       60.000
      300.000

     4000.000
       20.000
      315.000
```

```
 9820.605  - Z, Altitude, Bottom First
  179.315  - j                  Element.
   15.608  - h
    0.000  - i
  179.315  - J
   15.608  - H
    0.000  - I

  180.000  - ΣK, Cable Length
    3.600  - ΣW, Cable Weight
   25.793  - Wind, knots
  185.379  - Azimuth of Wind
  180.000  - Σβ + β_B
    5.379  - α
23530.616  - Reynolds Number
    1.200  - C_D
    1.799  - q, Dynamic Pressure
   40.488  - D_T, Total Element Drag
    3.513  - D_W, Vert. Drag Component
   40.158  - D_H
    3.701  - D_S

    0.679  - β_1
   84.271  - θ_1
 3196.674  - T_1

 9641.584  - Start of next element
  179.101           printout
   17.966
    0.213
  350.416
   33.634
    0.213

  360.000
    7.200
   28.584
  190.752
  180.679
   10.073
25191.772
    1.200
    2.055
   46.244
    4.545
   45.311
    8.049

    1.265
   83.446
 3193.433
```

```
4014.460
  10.579
   1.145
  14.518
5985.540
1475.075
2986.460

7272.000
 145.440
  20.289
 314.892
 265.492
  49.399
17549.267
   1.200
   0.898
   2.019
   1.063
   1.117
   1.304

   0.028
  35.963
3271.396

4003.890
  10.571
   1.138
  14.525
5996.110
1476.213
3000.984

7290.000
 145.800
  20.078
 314.971
 265.520
  49.450
17375.023
   1.200
   0.880
   1.979
   1.041
   1.094
   1.279

   0.028
  35.932
3271.459
```

```
OH SURF
ALT
        4003.890
HT
        5996.110
TENS
        3271.459
C.ELEV.ANG
          35.932
LENGTH
        7290.000
WT
         145.800
V.D
        1122.263
H,I,L
        1476.213
        3000.984
        3344.415
AZ.TO BLN
          63.807
B.EL.ANG
          60.849
S.R
        6865.745
C.AZ
          85.548
X +E
        3000.984
Y +N
        1476.213
OPT.ENT
0-NEW PROB
2-LOW CYCLES
```


77.007P
-I
+H,+I
N=500ft
N=500ft
-H
K
Z-T
H-I
Z-θ
Z-q
M=500lb
N=500ft
WINCH
Z-H
Z-I
Z
N=500ft
-H,-I
+I

In the printouts above, Case A shows the input data for a balloon at 10,000 ft, left side of the first page which is followed immediately with the surface or winch data on the right side of the page. At the end, an Option 2 or lower altitude cycle rerun mode was selected from the two choices provided. On the next page, Runs 2, 3, and 4 are shown. Run 2, for example, indicates the balloon is at 8500 ft with a wind azimuth of 225°. The same values of F_T and θ as in Run 1 were entered; an approximation since F_T and θ change with altitude and wind speed. Computation then commences leading directly to the surface or winch conditions. Run 3 automatically follows. Since 6000 ft is the last altitude above the surface in the wind field table (in Run 1), the Run 4 for this altitude is the last of the possible rerun cycles. The program therefore terminates by going back to the start, reprints the title, and is ready for a new problem.

On the next page, Case B, the same balloon problem is used in the basic form of the program where all parameters are printed at intermediate altitudes between the balloon and surface. The altitudes are determined by the location of the bottom end-point of each cable element and are therefore a function of the element length selected and the elevation angle of each element. On the right side of the page, the parameters for an altitude of 9820.685 ft, the bottom of the first element, are shown followed by 9641.584 ft for the second element. On the next page, the last two intermediate points are shown before the final surface or winch condition printout. These final winch figures agree with those in Case A, Run 1, since all input data for both cases was identical.

The plot for this problem is then shown. All notations are added; none are produced by the program except the point symbols noted in Section 3.5.1. Tick-mark intervals of 500 ft for all spacial dimensions and 500 lb for the tension were selected as part of the input data. A tension rounding factor of 3 made the initial F_T round from 3200 lb to 3000 lb thereby placing the 3000-lb tick mark in the middle of the upper (tension axis). The 3200 lb starting value may be seen to lie at the correct location to the right of the 3000-lb tick.

Because of the unusual cable properties selected to illustrate large cable rotation (cable diam = 1.25 in. and weighing only 20 lb per 1000 ft), the tension tends to remain constant and then increases above its starting value as it approaches the winch. In the more typical heavy cable used in tethered balloon work, the tension usually decreases with decreasing altitude.

The winds changing from south past westerly moving down from the balloon produced a large amount of cable turning as may be noted in the H-I plot which is really an X_B-Y_B plot looking vertically down from above the balloon. Due to the input value of wind azimuth at the balloon, 180°, the north direction is vertical along the H-axis toward the top of the paper. The balloon, always pointing down the paper at the intersection of the H-I axes, is "looking" into a south wind. The

westerly winds on the cable below the balloon, coming from the left, "turn" the cable in that direction. The end of the H-I plot represents the winch location. Final values of location, azimuth of the balloon from the winch, and cable out azimuth may be scaled from the plot and seen to agree with the printed figures. The two vertical projections of the cable position, Z-H and Z-I, can be seen to be consistent with the H-I plot since all three are at the same scale.

The cable elevation angle, θ, decreases from 85° at the balloon to 35.9° at the winch. In the full-size 10-in. × 15-in. plot, θ = 90° is 9 in. to the right of the Z axis. Knowing that the two vertical axes are 8 in. apart on the full scale plot, it is possible to ascertain the scale of θ on any plot reproduced to a smaller size. Similarly, since the scale of q is 2 lb/ft^2 per in. on a 10 in. × 15 in. plot, the scale on a plot of reduced size may be ascertained by reference to the distance between the two vertical axes.

Note that the dynamic pressure, q, is based on the wind velocity normal to each cable element and is therefore a function of the wind velocity squared, the atmospheric density, the elevation angle, θ, and the wind incidence angle, α. In this particular example, q reaches a maximum at 6000 ft where the wind is a maximum. If for example, the cable elevation angle at this point were 20° instead of an apparent 52°, the dynamic pressure would be reduced due to the smaller wind vector normal to the cable.

4. PROBLEM SOLUTIONS

Some examples of the solution to various tethered balloon cable problems will serve to indicate; (1) the ability to handle some of the intricacies involved in working with azimuth angles in a three-dimensional solution, and (2) the usefulness in advanced design efforts. Discussions here on the use of Program No. 77.007 apply equally to No. 77.007P.

4.1 Problems for Testing Program 77.007 and 77.007P Operation

4.1.1 180° AZIMUTH AMBIGUITY—TESTS 1 AND 2

A. <u>Test 1</u>

A sample problem used in Reference 1, Program No. 76.006 will be utilized here as Test No. 1. In this example of a two-dimensional problem, the wind was reversed 180° in direction between altitudes of 10,000 and 8000 ft by changing the sign of the wind magnitude in the wind profile entries in 76.006. In the case of this type of two-dimensional problem operated in the Program No. 77.007, the same reversal was first entered by a change in azimuth as shown before.

User Entries	76.006	77.007
Balloon Altitude, ft MSL	14,000	14,000
Surface Altitude, ft MSL	4,000	4,000
Internal C_D Computations	π (Yes)	π (Yes)
Cable Diameter, in.	0.28	0.28
Cable Weight/100 ft, lb	25.0	25.0
Element Length (K), ft	500.0	500.0
Balloon Total Force, lb	1385.0	1385.0
Total Force Elev. Angle, deg	79.4	79.4
Wind Field		
Z	14,000	14,000
Wind, knots	25	25
Azimuth, deg	——	180
Z	10,000	10,000
Wind, knots	60	60
Azimuth, deg	——	180
Z	8,000	8,000
Wind, knots	-15	15
Azimuth, deg	——	0
Z	5,000	5,000
Wind, knots	-30	30
Azimuth, deg	——	0
Z	4,000	4,000
Wind, knots	-20	20
Azimuth, deg	——	0

The final surface output parameters common to both programs were in agreement except for one particular aspect. In the two-dimensional program, the wind direction reversal takes place only in the sense that the wind velocity decreases from positive through zero to negative values. Thus the program retains the single vertical plane containing the cable, a horizontal plot shows a straight-line projection of the cable, and while not computed or printed, a value of I equal to zero is inferred. Figure 6 illustrates the straight line obtained in an X-Y (H-I) plot of the 76.006 output.

Also shown is the 77.007 output which has a final value of I equal to -719 ft.. This is caused by the rotation of the wind azimuth from 180° at an altitude of 10,000 ft to 0° at an altitude of 8000 ft. Thus the cable experiences a side force in this region and turns in azimuth. As can be noted in Figure 6, most of the bend is completed at 8000 ft altitude and no curvature in the horizontal plane occurs from that point to the surface. This effect is correct for the conditions specified by the particular way in which the wind was entered. If a south wind diminishing from 60 knots to zero followed by a north wind increasing from zero to 15 knots is the true condition, then additional points should be specified. If a single point at 8400 ft, 0 knots at 180° were specified (this is the zero wind velocity intercept between 10,000 and 8000) no bend would occur above 8400 ft. However, a bend would then take place between 8400 ft and 8000 ft if an end of a cable element fell within that area.

Therefore to reduce the probability of any bend, two sets of entries 1 ft apart is suggested for these cases where all winds are in the same or opposite direction. For this case, they would be:

Z	8400
Wind	0
Azimuth	180
Z	8399
Wind	0
Azimuth	0

When these are included in the wind table and a run is made, there is exact agreement with the two-dimensional program; that is, no side displacement occurs.

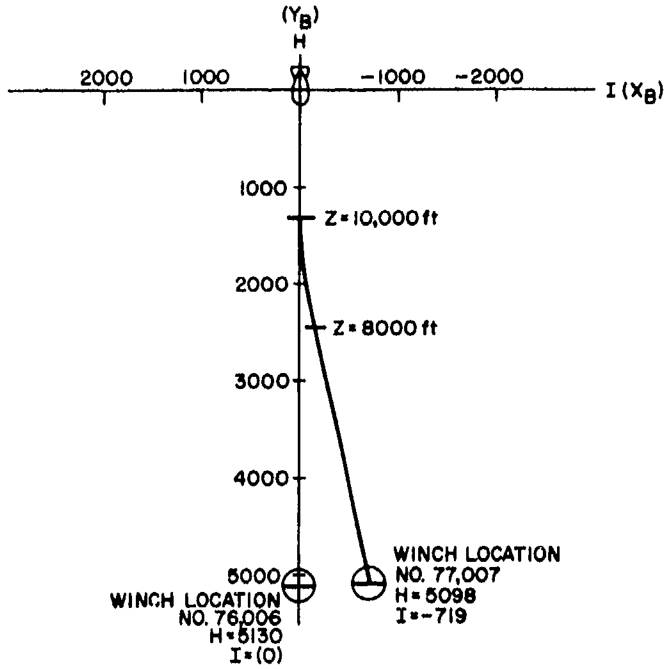


Figure 6. Plan View of Cable, Balloon Origin, X_B-Y_B Axes, Test 1

B. Test 2—180° Azimuth Rotation

In Test 1 the cable moved to a minus value of I when a wind azimuth interpolation between 180° and 0° was called for between 10,000 ft and 8000 ft. This indicates that counterclockwise rotation of wind azimuth will occur when an exact 180° turn is encountered. To show this, the following problem was established.

Balloon Altitude, ft MSL	10, 000
Surface Altitude, ft MSL	4, 000
Internal C_D Computations	π (Yes)
Cable Diameter, in.	0. 28
Cable Weight/1000 ft, lb	25. 0
Element Length (K), ft	500. 0
Balloon Total Force, lb	2000. 0
Total Force Elev. Angle, deg	85. 0

Wind Field	Test 2A	Test 2B	Test 2C
Z	10, 000	10, 000	10, 000
Wind, knots	25	25	25
Azimuth, deg	270	270	270
Z	9000	9000	9000
Wind, knots	50	50	50
Azimuth, deg	90	89	91
Z	4000	4000	4000
Wind, knots	25	25	25
Azimuth, deg	90	89	91

The program will always assume a rotation of wind azimuth between specified input points to the less than 180° direction. In Figure 7, the H-I plot shows that the cable responds by a move to the right (-I) from balloon to winch indicating a wind from the right quadrant for both Tests 2A and Test 2C. Therefore, when an exact 180° reversal is presented (2A), the data indicates that the wind rotation will be counterclockwise the same as in 2C where, between 10, 000 and 9000 ft, a 179° rotation from 270° through 180° to 91° is known to occur. In Test 2B the rotation of 179° would proceed from 270° through 0° to 89° or retain winds from the left quadrant in Figure 7. In this way, very different winch locations are indicated.

In summation, this demonstration shows the desirability of never specifying two adjacent wind azimuths exactly 180° apart. If there is any knowledge that would aid in better defining the direction of rotation an intermediate point should be part of the wind field input.

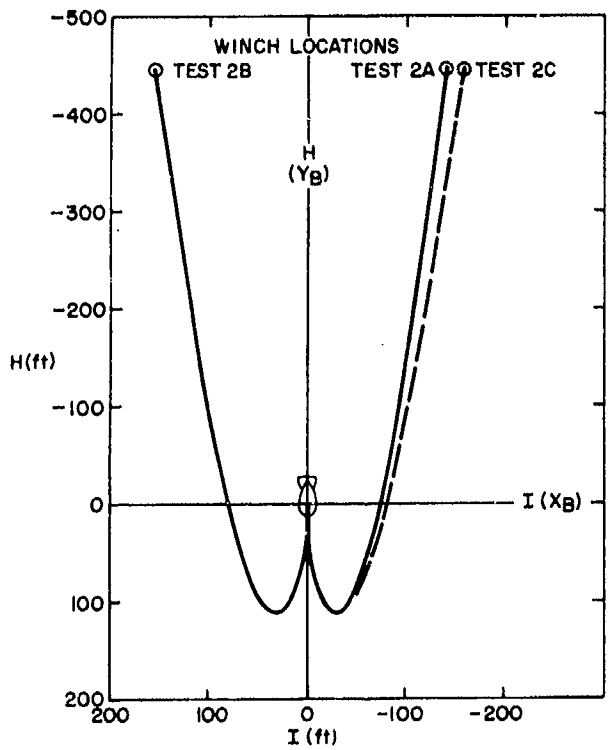


Figure 7. Plan View of Cable Balloon Origin, X_B-Y_B Axes, Test 2

4.1.2 EFFECT OF SELECTED VALUE OF ELEMENT LENGTH—TEST 3

Unless a value for the length of the cable element, K, used in the computations as specified, a value equal to 1 percent of the height or difference between the balloon and surface altitude is program-selected. In order to determine a reasonable value of K such that the number of program interactions and running time be minimized, a series of runs was made with K varying between 0.1 of 1 percent to 10 percent of the height. While this was done for only one specific problem—height, balloon force, cable specifications, and wind field—it does indicate something about the desirable order of magnitude of the K value.

Test 3 consisted of the following entries:

Balloon Altitude, ft MSL	14,000
Surface Altitude, ft MSL	4,000
Internal C_D Computations	π (Yes)
Cable Diameter, in.	0.28
Cable Weight/1000 ft, lb	25.0
Element Length (K), ft	Varies, see below
Balloon Total Force, lb	3,200.0
Total Force Elev. Angle, deg	85.0

Wind Field

Z	14,000	Z	10,000
Wind, knots	25	Wind, knots	40
Azimuth, deg	180	Azimuth, deg	300
Z	13,000	Z	8500
Wind, knots	25	Wind, knots	30
Azimuth, deg	225	Azimuth, deg	270
Z	11,000	Z	4000
Wind, knots	35	Wind, knots	15
Azimuth, deg	270	Azimuth, deg	210

K Values

ft	1000	500	400	200	100	50	10
% of Z_B-Z_S	10	5	4	2	1	.1	.1

Each of the K values was used in a run with three different printouts. Magnetic program cards were made up in three different forms as follows:

(a) Print of final output at the surface only,

(b) Print of Z, H, and I at end of each cable element plus final output at the surface,

(c) Complete print of program output as written—all parameters at end of each cable element plus final output at the surface.

The time required to run; (a) each of the 21 combinations without plotting (77.007), and (b) some combinations with plotting (77.007P) was also measured. Table 3 presents the values of several computed parameters. The values for a K value of 10 ft (0.1 of 1 percent of Z_B-Z_S) were taken as having zero error and were used as a base for error determination for other K values. These errors, shown in Figure 8, indicate that a K value of approximately 3 percent of Z_B-Z_S would assure less than 1 percent error in most of the parameters. The lack of smoothness in the error curves can be attributed to the manner in which the final surface altitude is reached. A function of the K length and cable elevation angle, the surface intercept can show somewhat random error. This in turn can introduce additional smaller errors in other parameters.

However, the general trends are indicative of the error in using too large a K value such as 10 percent of Z_B-Z_S.

Table 3. Output Parameters- Test 3

K, %(Z_B-Z_S)	.1	.5	1	2	4	5	10
K, ft	10	50	100	200	400	500	1000
Surf.Alt. ft	4000.000	4000.027	4000.518	4019.508	4018.151	4046.726	4091.463
Tension lb	2950.165	2950.404	2950.671	2951.693	2952.757	2954.018	2957.966
C.Elev.Angle	78.966	78.969	78.972	78.986	79.008	79.040	79.176
C.Weight lb	252.525	252.500	252.500	252.000	252.000	251.250	250.000
C.Length ft	10101.000	10100.000	10100.000	10080.000	10080.000	10050.000	10000.000
V.Drag lb	39.667	39.434	39.142	38.498	37.238	36.435	32.480
H ft	1019.415	1017.080	1014.246	1005.997	993.454	982.201	933.069
I ft	828.031	824.742	824.648	821.078	819.886	814.216	803.538
L ft	1311.443	1309.447	1307.124	1298.533	1288.085	1276.799	1230.726
AZ to Bln, dg	38.984	39.038	39.110	39.221	39.432	39.658	40.699
B.Elev Ang.	82.529	82.539	82.553	82.587	82.647	82.696	82.920
Sl.Range ft	10085.540	10084.548	10084.553	10064.611	10064.615	10034.707	9984.678
C.Azimuth dg	52.982	53.031	53.072	53.100	53.453	53.630	54.012

Figure 8 also indicates that the optional built in 1 percent K selector probably assures that all parameters are within 1 percent of true values, although, as next indicated, running times may be excessive for certain combinations of operation. Figure 9, presents the program running times as a function of the cable element length, K. If a K value of 3 percent of Z_B-Z_S were selec.ed from error considerations, the following running times would be indicated.

(a)	No Plot,	Only Final Print	2 min
(b)	No Plot,	Z, H, I Intermediate Alt Points and Final Print	2-3/4 min
(c)	No Plot,	Complete Intermediate Points and Final Print	7 min
(d)	Plot,	Only Final Print	5 min
(e)	Plot,	Z, H, I Intermediate Alt Points and Final Print	6 min
(f)	Plot,	Complete Intermediate Points and Final Print	9-3/4 min

It should be emphasized that these errors and running times apply to the particular problem used in this model. W: l· ·y differing problems, such as more variations in the wind azimuth or greater wind magnitude producing larger—or smaller cable elevation angles at the surface—will have a different set of errors and of course, running times.

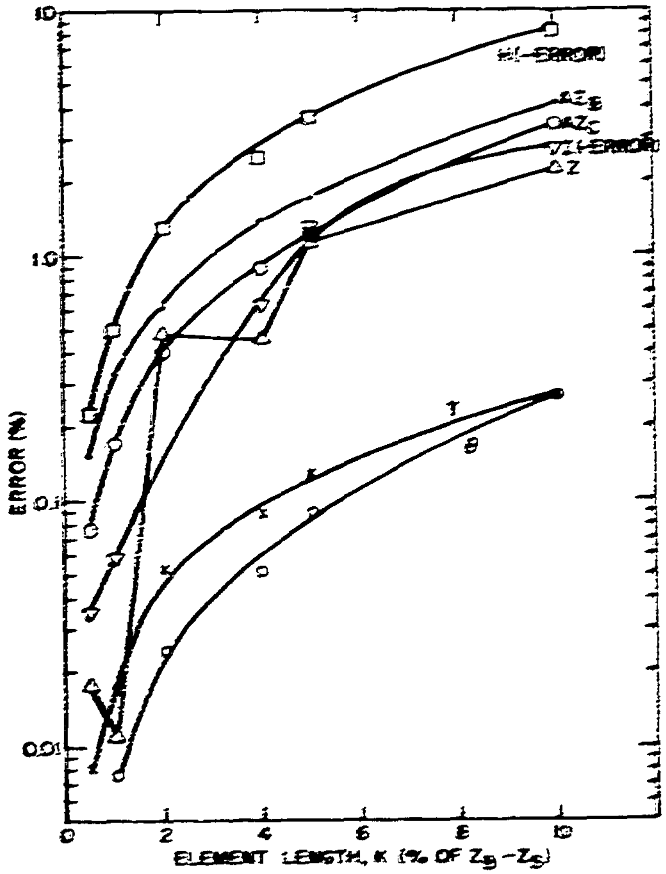


Figure 8. Errors of Various Final Output Parameters as a Function of Cable Element Length, Test 3

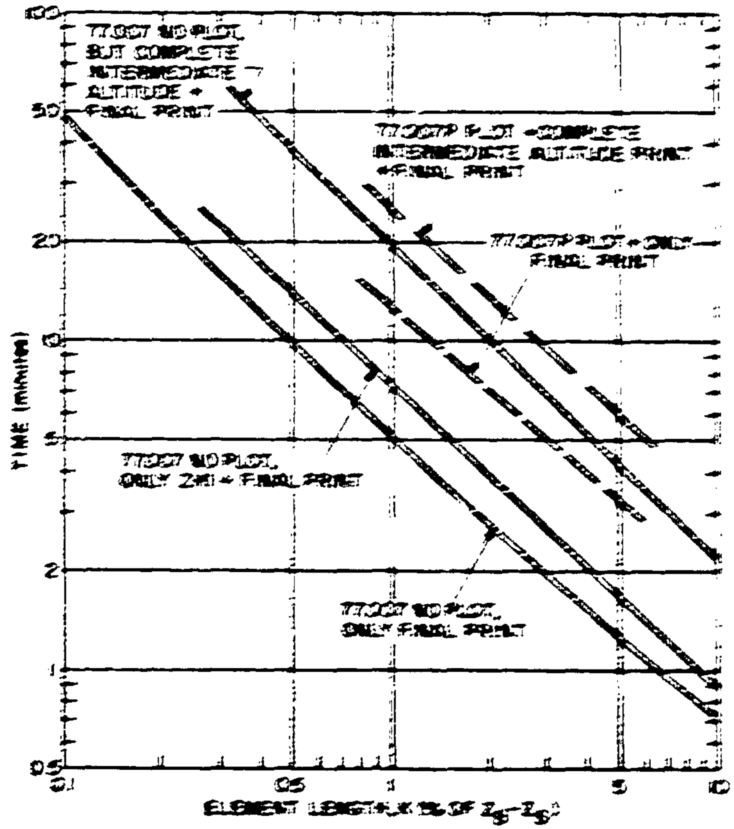


Figure 9. Program Running Times, Test 3

4.1.3 SOLUTION CHECK FOR ALL BALLOON POINTING AZIMUTHS—LARGE CABLE—TESTS 4 THROUGH 12

The cables specified in previous tests have small diameters. They are typical of the type of cables selected for tethered balloon operation with a high strength-to-weight ratio so that small diameters are possible thereby minimizing drag loads as well as cable weight. With small drag loading, the cable is less sensitive to winds and in particular, winds from the side will not produce large deflections easily shown on an X-Y plot. In order to clearly show large movement of the cable and illustrate that this program can properly handle large cable turns, a peculiar type of cable will be specified in this group of tests. In addition, the wind field will include some large wind speeds.

A large cable diameter of 1.25 in. together with winds of large magnitudes will produce significant drag. A cable weight of only 20 lb per 1000 ft insures that cable weight will not predominate in the calculations of cable deflections. (This is not a real cable.) Other input data include:

	Z_B	10,000 ft MSL
	Z_S	4,000 ft MSL
Internal	C_D	π (Yes)
	K	180 ft
	F_T	3,200 lb
	θ	85 deg

and the following wind profiles:

Altitude ft MSL	Wind Speed knots	Azimuth, deg Test 4	Test 5	Test 6	Test 7	Test 8	Test 9	Test 10	Test 11	Test 12
10,000	25	180	180	180	270	270	0	0	90	90
8,500	40	180	225	135	315	225	45	315	135	45
7,000	50	180	270	90	0	180	90	270	180	0
6,000	60	180	300	60	30	150	120	240	210	330
4,000	20	180	315	45	45	135	135	225	225	315

Test 4 is a two-dimensional problem since the complete wind profile shows wind from the south (180°). The balloon therefore points towards 180° and the winch lies south of the balloon. Hence the azimuth from the winch to the balloon and the azimuth of the cable leaving the winch should be 0° as was computed (see Table 4). The same problem could be run in Program 76.006 since it is a two-dimensional condition.

Table 4. Output Parameters—Tests 4 Through 12

Test No.	Surface Altitude ft	Height ft	Tension, Winch lb	Cable El. Angle deg	Cable Length ft	Cable Weight lb	Tot.Vert. Drag lb	Baln.El. Angle dog	Slant Range ft	Distance H ft
4	4005.25	5994.75	3124.00	28.89	8046	160.92	1517.72	52.20	7586.43	4649.39
5-12	4003.89	5996.11	3271.11	35.93	7290	145.80	1122.26	60.85	6865.74	1476.21

	Distance I ft	Distance L ft	Azimuth to Balloon deg	Cable Out Azimuth deg	X + East ft	Y + North ft
4	0	4649.39	0	0	0	4649.39
5	3000.98	3344.42	63.81	85.55	3000.98	1476.21
6	-3000.98	3344.42	296.19	274.45	-3000.98	1476.21
7	3000.98	3344.42	153.81	175.55	1476.21	-3000.98
8	-3000.98	3344.42	26.19	4.45	1476.21	3000.98
9	3000.98	3344.42	243.81	265.55	-3000.98	-1476.21
10	-3000.98	3344.42	116.19	94.45	3000.98	-1476.21
11	3000.98	3344.42	333.81	355.55	-1476.21	3000.98
12	-3000.98	3344.42	206.19	184.45	-1476.21	-3000.98

While the wind speed profiles for Tests 4 through 12 are all the same, the azimuth profiles of Test 5 through Test 12 indicate winds turning in azimuth as the altitude decreases. In Tests 5 through 12 there is essentially the same degree of turning in order to represent the balloon pointing in 4 different directions with the cable subjected to winds approaching 90° from either side of the cable. The wind azimuths in Tests 6, 8, 10, and 12 are mirror images of those in Tests 5, 7, 9, and 11 respectively. They were selected in order to show positive and negative values of the displacement, I, and to produce a wide range of cable and balloon azimuth angles and X and Y values on the surface. (Section 3.5.7 contains the tape output for Test 5 only.) Table 4 contains the final output values to illustrate; (1) parameters such as tension, cable elevation angle, cable length, slant range, etc., are identical for all tests as they should be; and (2) that balloon and cable azimuth angles and the geographic-axis displacements X and Y differ as expected.

Figure 10 illustrates the eight H-I or X_B-Y_B plots of these tests to show that the parameters—azimuth to the balloon from the winch and the azimuth of the cable leaving the winch—are consistent, and therefore properly handled through the 0° to 360° boundary.

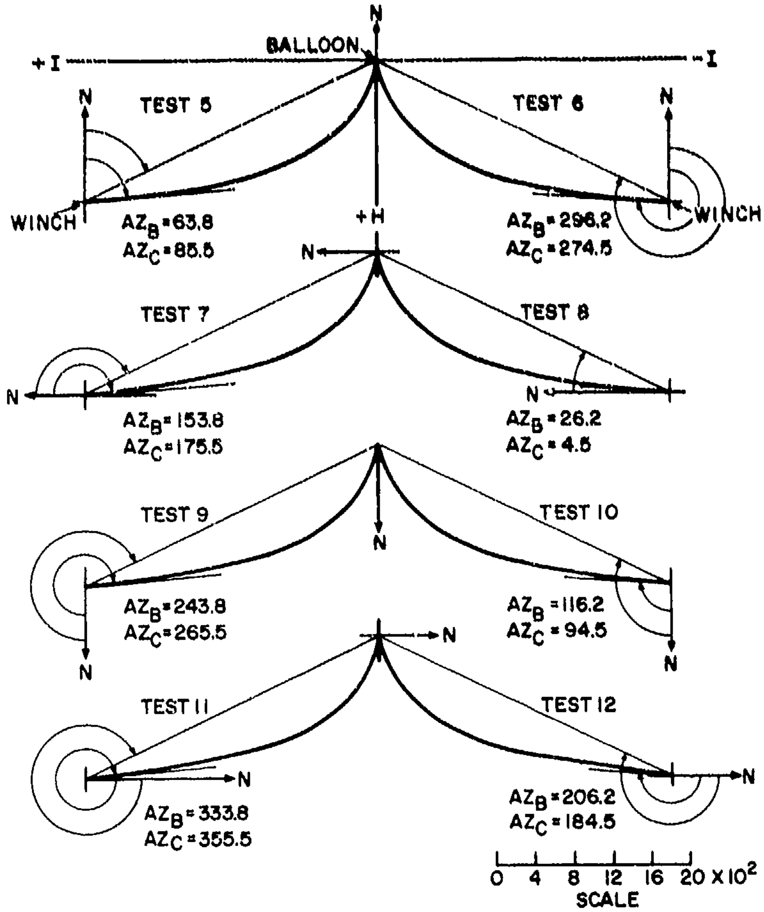


Figure 10. Plan Views of Cables, Balloon Origin, X_B-Y_B Axes, Tests 5 Through 12

4.1.4 TRENDS IN CABLE ROTATION DUE TO WIND AZIMUTH ROTATION—LARGE CABLE—TESTS 13 THROUGH 18

The balloon and cable specifications and the magnitude of the wind used in Tests 4 through 12 were retained in Tests 13 through 18. As shown in Figure 11, the azimuth angles of the wind on the cable were varied; (a) in Tests 13 through 16 to greater amounts of clockwise rotation than Test 5, and (b) in Tests 17 and 18 through less severe rotation clockwise then counterclockwise.

The H-I (X_B-Y_B) plot in Figure 12 indicates the tightness of the cable turn produced by very severe wind azimuth rotation. When Figure 12 is transferred in Figure 13 to a common winch set of axes, X_W-Y_W, a clearer picture of the balloon movement with variations in wind rotation on the cable is possible. Because the balloon in these particular tests is pointing exactly south (180° azimuth), the X_W-Y_W axes in Figure 13 are also the X-Y or geographical axes with North pointing

up the Y-axes. It can be seen that the greatest amount of wind rotation (Test 16) places the balloon closest to the winch in this horizontal projection of the cable geometry. The winch/balloons in Tests 17 and 18 and in Test 4, the two-dimensional case, are outside the boundaries of Figures 12 and 13. The length of the cable required in each of the tests, as shown below, decreases with proximity to the winch as projected in the horizontal plane.

Test Number	Horizontal Distance Balloon to Winch	Length Cable
4	4649	8046
18	4445	7920
17	4285	7812
5	3344	7290
13	2425	6786
14	1192	6300
15	828	6210
16	515	6120
Min Possible C. Length $Z_B - Z_S$		6000

The two-dimensional case, winds all from the same direction (Test 4), produces the greatest displacement and cable length.

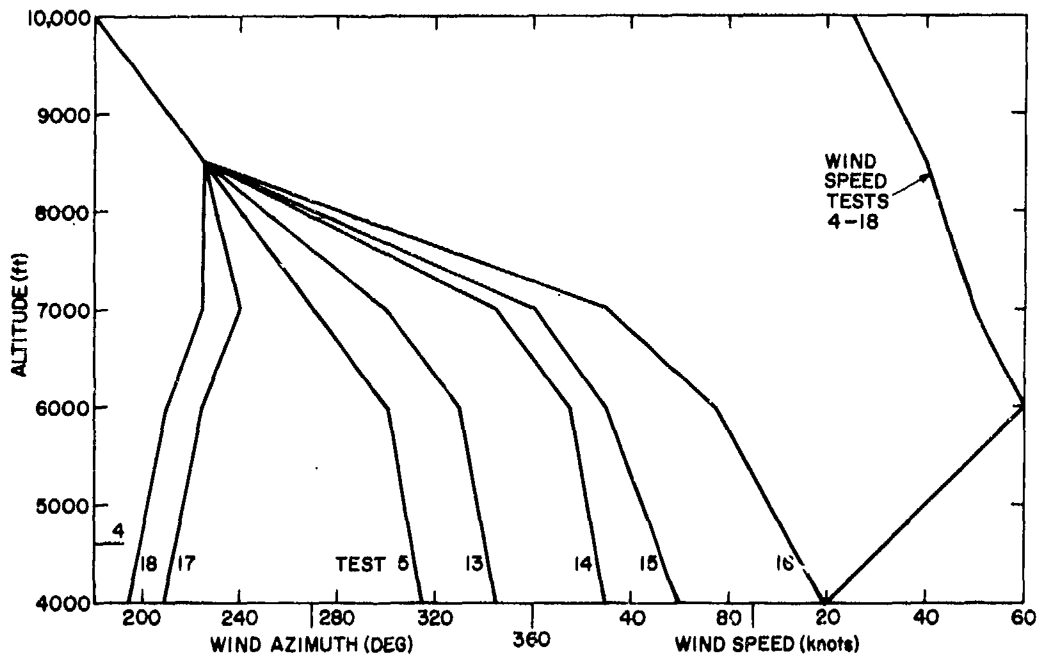


Figure 11. Wind Profiles, Tests 4, 5, and 13 Through 18

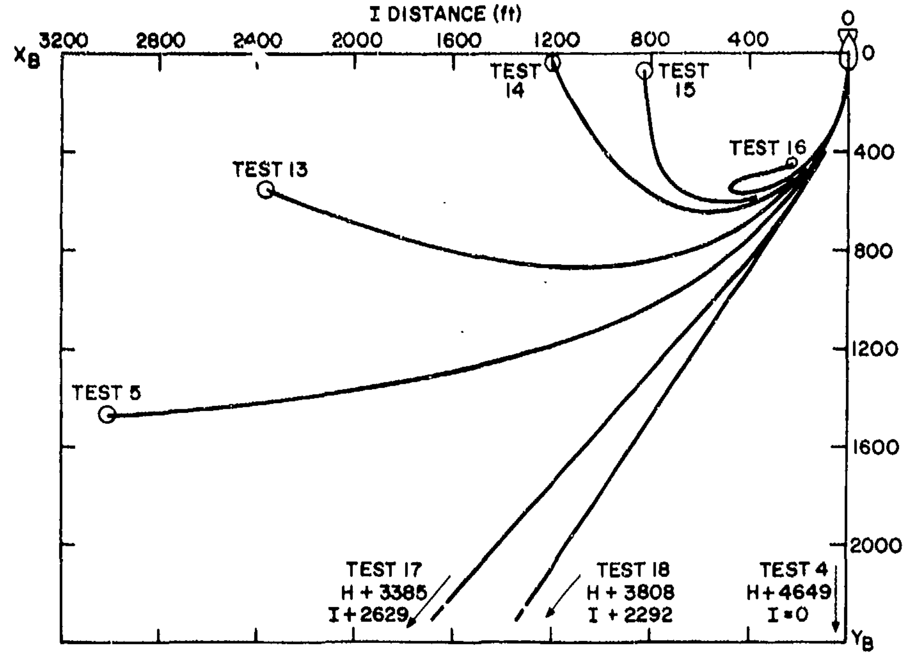


Figure 12. Plan View of Cables, Common Balloon Origin, X_B-Y_B Axes, Tests 4, 5, and 13 Through 18

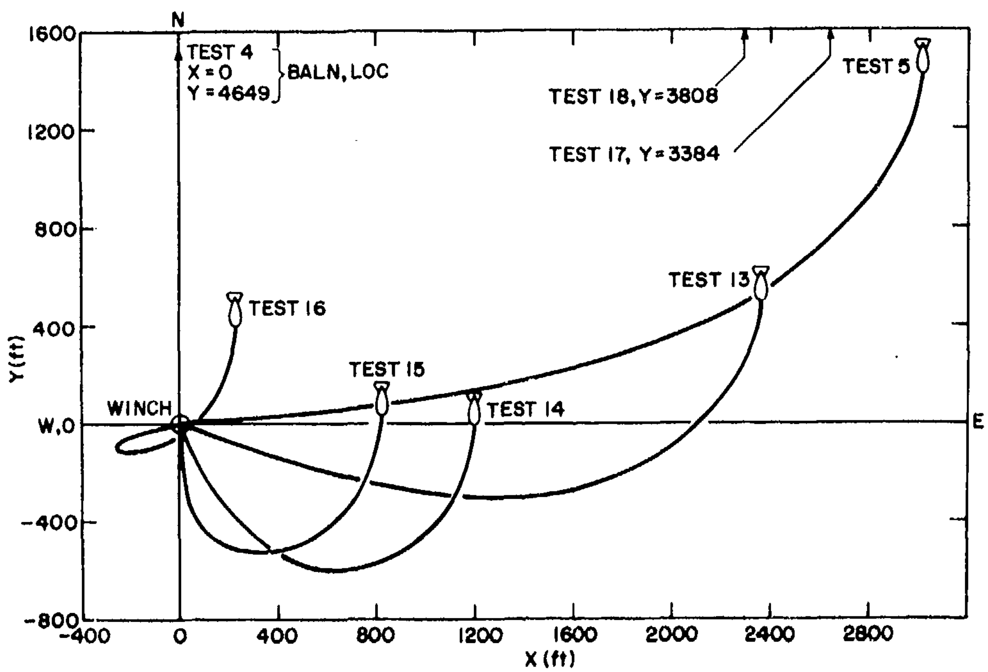


Figure 13. Plan View of Cables, Common Winch Origin, X-Y Axes, Tests 4, 5, and 13 Through 18

Figure 14 presents the variation of tension and elevation angle with altitude for some of these tests. Test 5 maintains the highest level of tension of all the tests. In this case the tension at the winch is greater than at the balloon. Its elevation angle at the winch is only 36°. While Test 16 produces a very vertically aligned cable (at 6000 ft it is nearly vertical) the tension decreases down the cable to a minimum at the winch. The more than 8000 ft of cable in Test 4 enters the winch at a very low 29° elevation angle. While these effects are all exaggerated cases due to the very lightweight large-diameter cable, they illustrate the necessity to make computations over a wide range of possible meteorological conditions, if precise information on cable behavior is wanted for many different types of flying days.

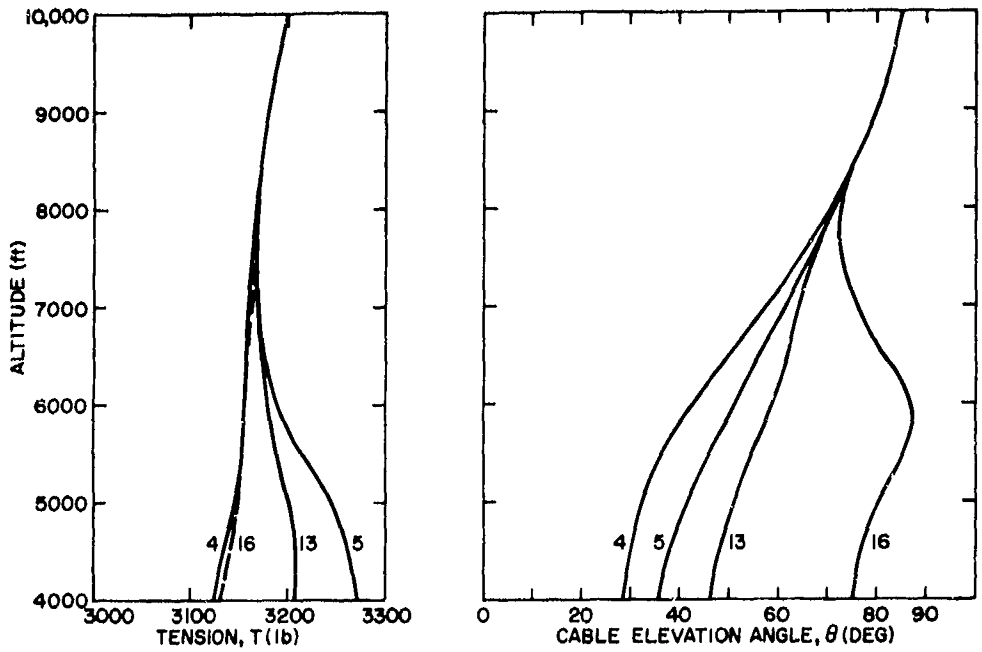


Figure 14. Cable Tension and Elevation Angle Variation with Altitude, Tests 4, 5, 13, and 16

4.1.5 EXTREME CABLE ROTATION WITH BALLOON IN ALL QUADRANTS—LOW ALTITUDE CYCLES—TEST 19

The following problem is introduced to illustrate; (a) a corkscrew-like cable geometry which extends through all 4 H-I quadrants, and (b) the use of a lower altitude cycles for static evaluation of the balloon during ascent or decent. Again, a

large diameter and extremely lightweight cable is utilized to obtain the exaggeration desired and is not an available product.

Max. Balloon Alt, Z_B	10,000 ft MSL
Surface Alt, Z_S	0 ft MSL
C_D	1.0
Cable Diameter	2.0 in.
Weight per 1000 ft	10 lb
Element Length K	250 ft
Balloon Total Force	3200 lb (also used at all lower alts)
Angle of Total Force	85°

Z	10,000	7000	4000	1000
Wind	25	40	40	40
AZ	180	45	180	315
Z	9000	6000	3000	0
Wind	35	40	40	40
AZ	270	90	225	0
Z	8000	5000	2000	
Wind	40	40	40	
AZ	0	135	270	

Note that the wind magnitude from 8000 ft to the surface is 40 knots and that a constant $C_D = 1.0$ is used in this problem.

Figure 15 is the H-I (X_B-Y_B) plot with the balloon at 10,000 ft showing that the cable extends in a spiral through all 4 H-I quadrants. Figure 16 shows the Z-H and Z-I vertical views of the cable (at a different scale than Figure 15) to show the tightness of the spiral.

Figure 15 was then converted to an X_W-Y_W plot, Figure 17a. Again, because the azimuth of the wind at the balloon at 10,000 ft is 180°, these axes can be considered as X-Y with North up the paper. The cable for Z_B = 10,000 ft is shown appearing smaller than in Figure 15 due to the reduced scale.

The X and Y values from the lower balloon altitude cycle runs (Option 2) were then plotted as points on Figure 17a. The wind azimuth at the balloon at each of the 1000 ft levels was used to "aim" the small balloons (drawn thereon) into the wind. Each is annotated with balloon altitude. As would be expected with the wind field established for this test, the balloon travels through a spiral path relative to the winch during ascent from the surface to 10,000 ft.

In the interests of clarity in Figure 17, the cable plan views are drawn only for balloon altitudes of 10,000, 8000, 6000, and 4000 ft. In spite of the widely dispersed balloon locations, the cable leaving the winch remains within a 45° to 155° range of azimuths at all balloon altitude, Figure 17b.

This type of exaggerated case also causes the cable tension to increase moving from balloon to surface, Figure 18a. At all lower balloon altitudes, the winch tension is larger than the starting tension, 3200 lb. The largest tension is at the

winch except for balloon altitudes of 5000, 6000, and 7000 ft where slightly higher values exist somewhere along the cable.

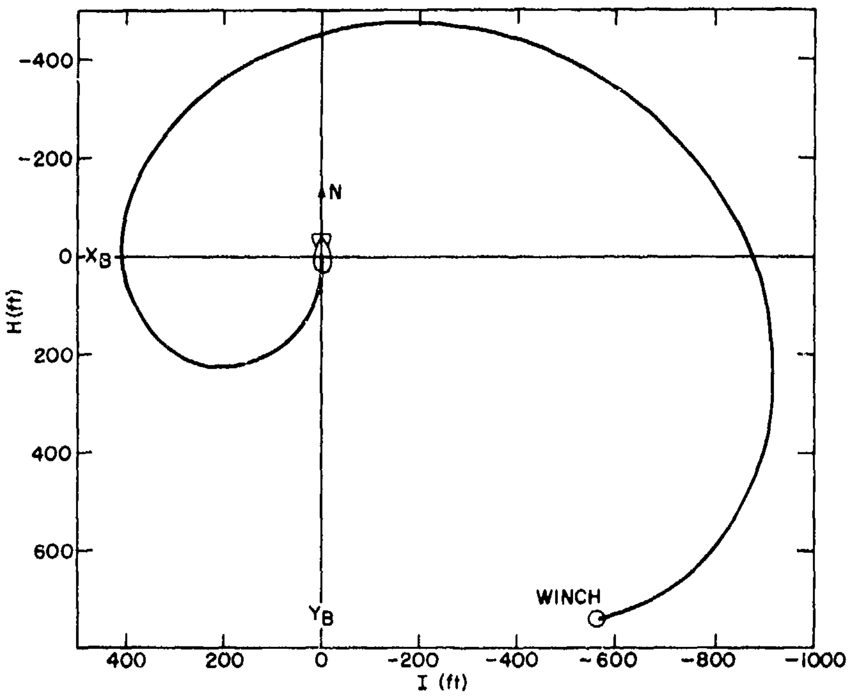


Figure 15. Plan View of Cable, Balloon Origin, X_B-Y_B Axes, Test 19

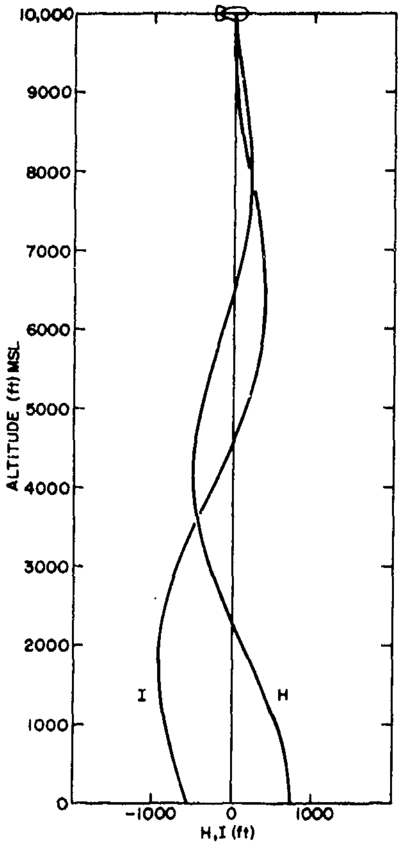


Figure 16. Vertical Views of Cable, Test 19

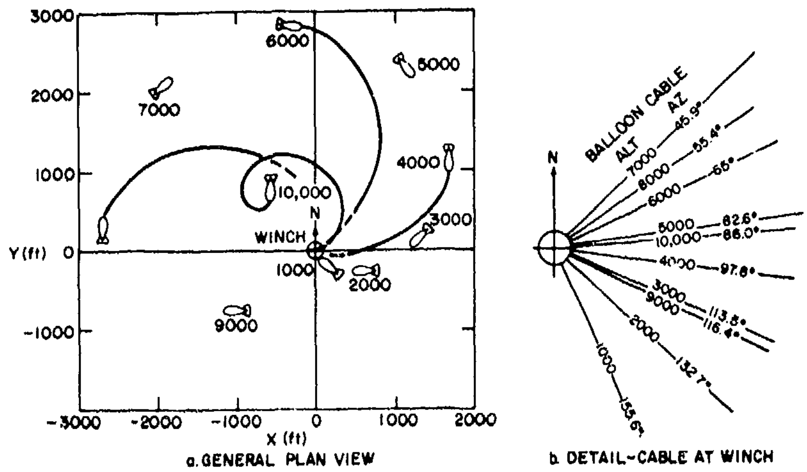


Figure 17. Plan View of Cable with Balloon at Various Altitudes, Common Winch Origin, X-Y Axes, Test 19

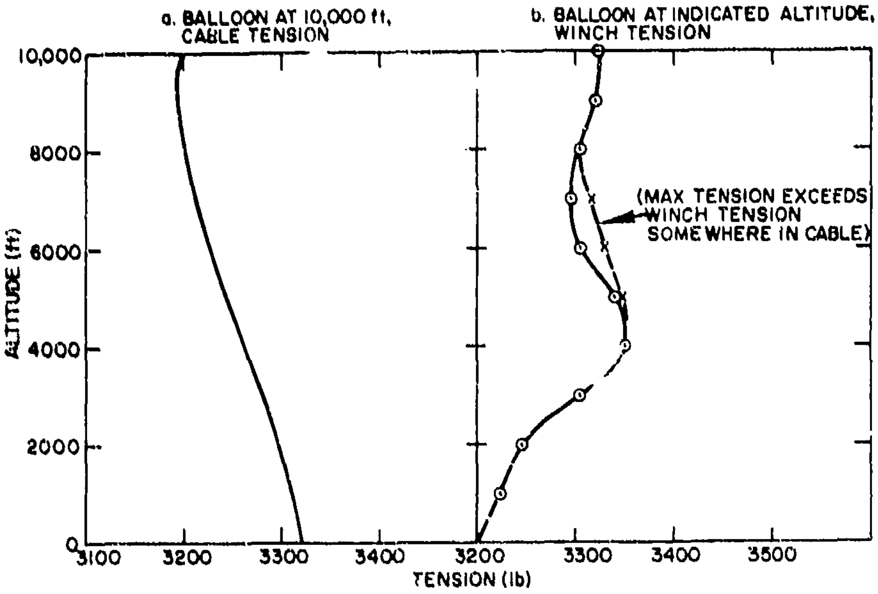


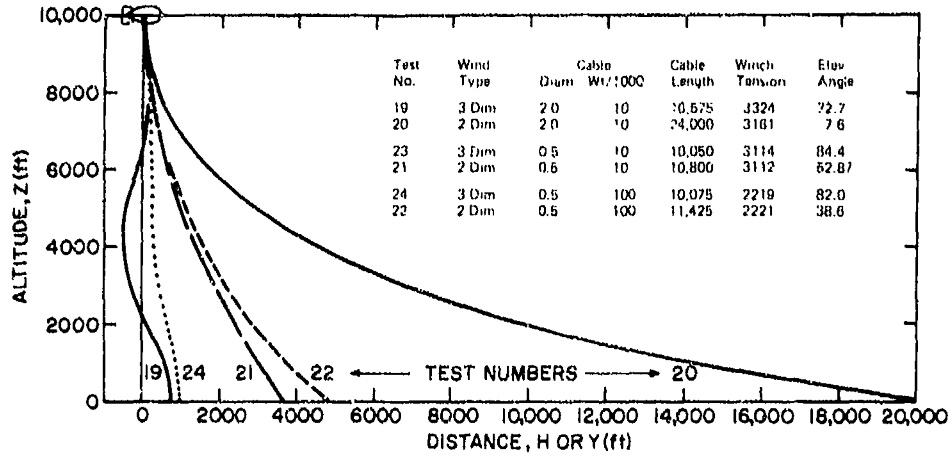
Figure 18. Tension—Test 19

4.1.6 CHANGES WHEN LESS EXTREME CABLES OR AZIMUTH ROTATIONS ARE INTRODUCED—TESTS 19 THROUGH 24

The problem in Test 19 used a severely rotating wind field and a large light-weight cable to illustrate a spiral rotation of the cable. Tests 20 through 24 retain the same wind magnitudes as in Test 19 but vary other parameters.

Test 20 uses the same cable but is a two-dimensional case with the winds at all levels from the same direction. In Figure 19, the effect may be noted as large increases in downwind displacement and in cable length and a very low cable elevation angle at the winch.

Tests 23 and 21 are three-dimensional and two-dimensional repeats of the same problem as Test 19 and 20 except with a smaller diameter cable (0.5 in.) to reduce the drag components. While winch tensions show a small decrease over Tests 19 and 20, the elevation angles are increased and the cable length, (in the two-dimensional case) is greatly reduced.



Test No.	Wind Type	Cable Diam	Cable Wt/1000	Cable Length	Winch Tension	Elev Angle
19	3 Dim	2.0	10	10,675	3324	72.7
20	2 Dim	2.0	10	24,000	3161	7.6
23	3 Dim	0.5	10	10,050	3114	84.4
21	2 Dim	0.5	10	10,800	3112	52.87
24	3 Dim	0.5	100	10,075	2219	82.0
22	2 Dim	0.5	100	11,425	2221	38.8

Figure 19. Vertical Views of Cable, Tests 19 Through 24

Tests 24 and 22 are three-dimensional and two-dimensional repeats of the same problems except that the cable is brought into a completely realistic specification by an increase in weight to 100 lb per 1000 ft with a diameter of 0.5 inches. In the comparison of two-dimensional cases, the cable length and down range displacement increase, but the winch tension is significantly reduced. In the comparison of three-dimensional cases, the winch tension is similarly reduced without much change in cable elevation angle.

Tests 23 and 24 both exhibit a different form of cable rotation in the plan view than Test 19. The effect of the smaller drag producing cable is similar in both Tests 23 and 24, therefore Figure 20 includes results for only Test 24. In the H-I plot—left side—the cable becomes vertical at some altitude and produces the sharp discontinuity shown thereon. This is the special case, discussed in Section 3.2.8b which required special computational handling within the programs. On the right side of Figure 20, a portion of the vertical-plane plots of Z-H and Z-I show that the cable goes vertical at an altitude of approximately 6600 ft. These comparisons indicate that a practical tether cable will rarely produce the spiral cable configuration in a wind field showing continuous rotation with changes in altitude. More likely is the form shown in Figure 20 with, in addition, a decrease in cable tension as one moves down the cable to the surface, as with Test 24.

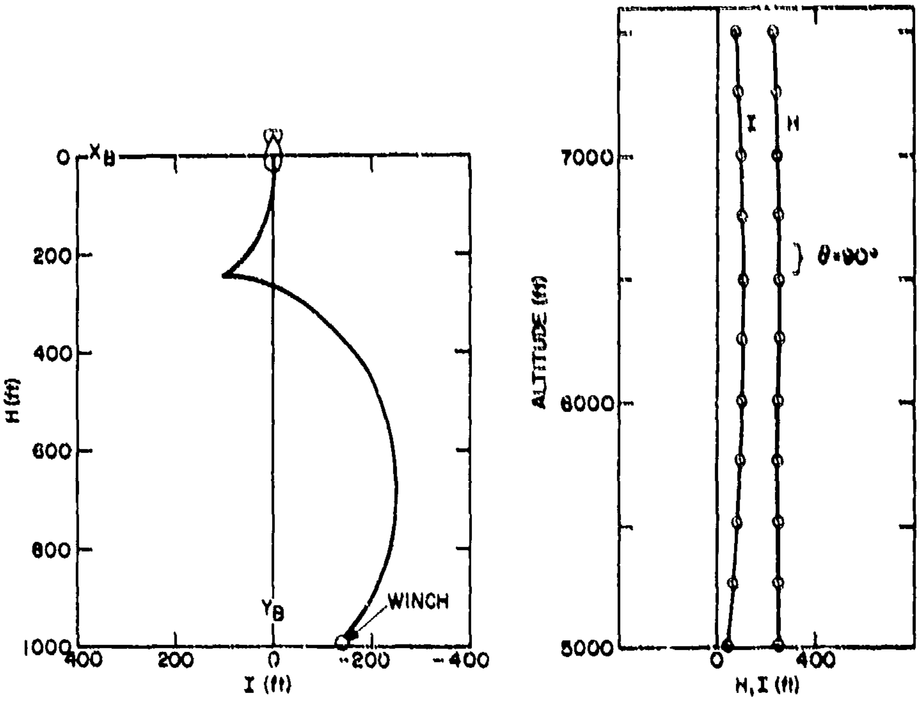


Figure 20. Plan and Vertical Views of Cable, Test 24

4.2 Problems for Testing Program 77.007B Operation

When Test 24 was completed in a Program 77.007 run, Option 3 was selected to call in Program 77.007B. This established a fixed cable length of 10,075 ft which was found in the Test 24 solution (Col. 1 below). The wind profile for Test 22, a two-dimensional case—wind all from same direction—with wind magnitudes the same as Tests 19 through 24, was introduced as an input into Program 77.007B. This test will therefore be designated as Test 24.22.

Some of the output is shown in Col. 2 below. It indicates that the balloon has descended from an altitude of 10,000 ft (Test 24) to 8980 ft (Test 24.22) in the presence of the two-dimensional wind. To indicate consistency within the programs, the altitude of 8980 ft was next entered in Program 77.007 and a solution made again with same cable and winds used with 77.007B. The surface output, Col. 3, exactly duplicates the Program 77.007B output. If a printout and/or plot of cable parameters vs altitude were required, this is the procedure to follow after a 77.007B solution.

	(1)	(2)	(3)
	Test 24 77.007	Test 24.22 77.007B	Test 22.22 77.007
Program No.:			
Input Data			
Balloon Alt.	10,000	—	8,980
Winds	3-dim.	2-dim.	2-dim.
Cable Length	—	10,075	—
F_T	3,200	3,200	3,200
θ	85	85	85
Output Data			
Balloon Alt.	—	8,980	—
Cable Length	10,075	10.075	10,075
Winch Tension	2,219	2,322	2,322
Cable Elev. Angle	82.0	42.8	42.8
Azim. to Baln.	352.0	0	0
Cable Azimuth	48.8	0	0
X	-139	0	0
Y	990	4,054	4,054

4.3 Practical Problems—High Altitude Tethered Balloon

Since the developments of these programs were predicated on their need in solving advanced tethered balloon problems, some illustrative cases are pertinent here. As described in Reference 1, there is an AFGL plan to tether a balloon at an altitude of 20 km (65,616 ft MSL). Two-dimensional cable problems and their solutions were presented in Reference 1, Section 5, utilizing Program No. 76.006.

4.3.1 FIXED BALLOON ALTITUDE

To illustrate the effect of winds having directions that vary with altitude on the cable design and system performance, one of the aforementioned two-dimensional problems will be repeated and expanded here into three dimensions. The basic problem using the design wind magnitude profile, the internal drag coefficient computations, and the same cable as shown in Reference 1, Table 3, Line 3 was selected as a typical high-altitude problem. Test 25, with its input shown below, is a repeat of that problem using wind azimuths at all altitudes equal to 270° to provide the comparable two-dimensional case in Program 77.007.

Input - Test 25

Z_B	=	65,616 ft	Z	=	65,616 ft	Z	=	36,000
Z_S	=	4400 ft	Wd	=	25 knots	Wd	=	26
Diam	=	0.3 in.	AZ	=	270 deg	AZ	=	270
Wt	=	25.0 lb/100 ft	Z	=	52,500	Z	=	29,500
K	=	1000 ft	Wd	=	15	Wd	=	18
F_T	=	3073 lb	AZ	=	270	AZ	=	270
θ	=	81.6 deg	Z	=	42,500	Z	=	5500
			Wd	=	27.5	Wd	=	9
			AZ	=	270	AZ	=	270
						Z	=	4400
						Wd	=	8
						AZ	=	270

The output parameters, shown in the first line of Table 5A agree with those of Reference 1. Tests 26 through 30 were then run with the same input as Test 25 except for wind azimuth. The azimuths, Figure 21, were widely varied from test to test; those used in Test 29 represent a greater than 360° continuous azimuth rotation between the balloon and the surface.

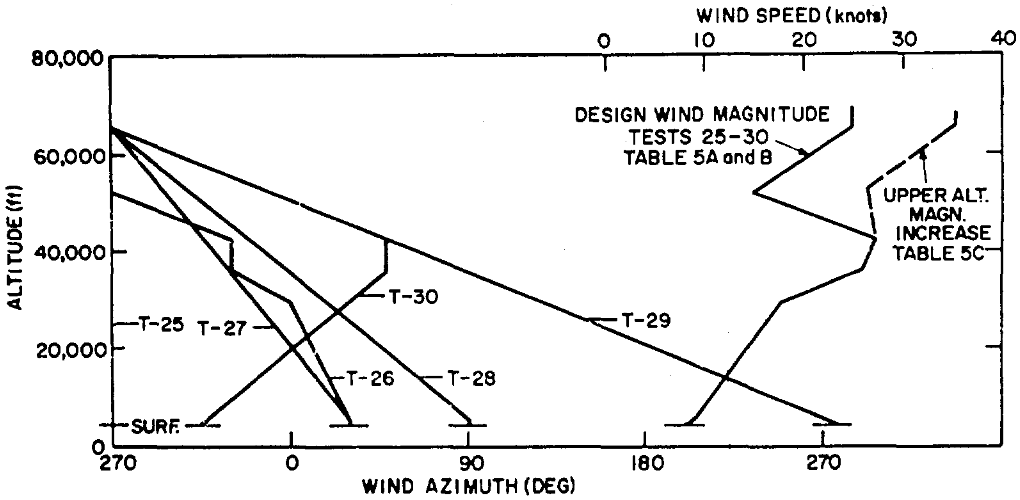


Figure 21. Wind Profiles, Tests 25 Through 30

In Table 5A it may be noted that the widely varying azimuth conditions introduced in Tests 26 through 30 produced little change in the tension at the winch but increased the cable elevation angle at the winch seemingly in proportion to the "severity" of the azimuth rotation. Cable length was decreased by small amounts but the balloon decreased its horizontal displacement from the winch by as much as 50 percent. For a given fixed profile of wind magnitude, the changes in wind direction act principally to improve (increase) cable elevation angle at the winch and to improve (decrease) balloon displacement.

Table 5. *High-Altitude Tether-Cable Test Data—Tests 25 Through 30*

Test No.	Bln Alt	Bln Wind	Bln AZ	Winch Tension	Cable Elev Angle	Cable Length	Azim to Balloon	Bln Elev Angle	Slant Range	Cable Az Out of Winch	Balloon Position from Winch X+East	Balloon Position from Winch Y+North
	ft	knots	deg	lb	deg	ft	deg	deg	ft	deg	ft	ft
A. BALLOON at 65616 ft, MSL. SAME WIND MAGNITUDE PROFILE, VARIOUS AZIMUTH PROFILES per Fig. 21												
25	65616	25.0	270	1547	54.3	65300	90.0	71.3	64633	90.0	20766	0
26	"	"	"	1548	62.9	64000	109.1	74.2	63577	126.8	16320	-5641
27	"	"	"	1548	61.0	64300	108.6	73.5	63815	124.3	17142	-5773
28	"	"	"	1544	69.6	63300	114.0	76.2	63021	135.5	13735	-6114
29	"	"	"	1545	77.3	62000	94.7	80.8	61964	81.1	9860	-814
30	"	"	"	1545	70.0	62600	119.0	78.9	62371	140.5	10467	-5795
B. CABLE LENGTH=62,600 ft-from Test 30. SAME WIND MAGNITUDE PROFILE, VARIOUS AZIMUTH PROFILES per Fig 21												
30.29	66160	25.0	270	1530	77.1	62600	94.7	80.7	62563	81.2	10078	-821
30.28	64996	24.5	272	1562	69.6	"	115.6	76.2	62326	136.8	13447	-6445
30.27	64116	23.9	273	1587	61.5	"	110.9	73.6	62153	125.9	16360	-6249
30.25	63316	23.2	270	1604	56.0	"	90.0	71.7	62032	90.0	19478	0
C. CABLE LENGTH=62,600 ft-from Test 30. UPPER ALT. WIND MAG. INCR., VARIOUS AZIMUTH PROFILES per Fig. 21. See text												
30.25	58260	30.5	270	1829	43.5	62600	90.0	60.5	61868	90.0	30444	0
30.30	60920	32.3	298	1767	51.5	"	135.8	65.4	62110	145.4	18057	-18557

For the condition of fixed balloon altitude—cable length varied to maintain the height—and fixed balloon wind magnitude, two conclusions seem to be in order.

(a) If the cable "reaches" the surface when any two-dimensional wind profile (constant azimuth) type of problem is solved, the tension at the winch will never be more than a few percent higher than that computed for a zero-wind case. In Reference 1, for the same balloon and cable, various wind magnitude profiles on the cable produced the same tension at the winch in spite of widely varying cable lengths, total cable weights, and drag forces. Therefore, the tension calculated with the simple no-wind (on the cable) relationship

$$T_W = F_T - \text{Wt of Cable} = F_T - (\text{Length} \times \text{Cable Density})$$

or

$$T_W = F_T - (Z_B - Z_S) \times \text{Cable Density}$$

or

$$T_W = 3073 - (65,616 - 4400) \times (25/1000) = 1542.6 \text{ lb}$$

is within 1 lb of the tension computed in Test 25.

(b) When in any two-dimensional wind profile type of problem the total vertical drag is found to be less than the total cable weight, a change to any azimuth profile (three-dimensional with same wind magnitude) will cause little significant change in the winch tension provided the cable "reaches" the surface. There are even some combinations of unusual cables and heights investigated (see Tests 4 and 5, Table 4) where the vertical drag may exceed the cable weight without the wind azimuth variations causing excessive adverse tension changes.

Increases in the wind magnitude at the balloon can of course, change the balloon total force and therefore, the winch tension and modify the above statements; as will be shown in Section 4.3.2b.

4.3.2 FIXED CABLE LENGTH

(a) Effect of Wind Azimuth Change

If the cable length, 62,600 ft found for the conditions specified in Test 30 above is held fixed, the winds (varying only in azimuth) from some of the previous tests may be introduced through 77.007B to determine resulting new balloon altitudes and cable conditions. Test 30.29 in Table 5B, for example, is a combination of the Test 30 cable length with Test 29 winds. The balloon total force and angle used in these tests are held fixed at 3073 and 81.6. This is admittedly imprecise but sufficiently close enough to illustrate trends in the system behavior that are due principally to changes in the wind azimuths.

Table 5B indicates the results of such runs. As expected, the balloon is moved down in altitude from the original 65,616 ft when the degree of rotation of the wind between the balloon and the surface is less severe than the "base" wind (Test 30). For the two-dimensional wind case (Test 30.25), the balloon drops 2300 ft in altitude. Its downrange displacement nearly doubles; 10,467 to 19,478 ft. The cable elevation angle at the winch is reduced from 70 to 56° with a tension increase of 59 lb. When a greater amount of rotation is introduced, the balloon rises in altitude as shown in Test 30.29; a constant 25 knot wind magnitude was held above the starting altitude of 65,616 ft.

(b) Effect of Wind Magnitude Change

Here the cable length, 62,600 ft, found with Test 30 conditions is again held fixed. Wind magnitude changes were made as follows:

Z, ft	Wind, knots	
	Original Tests 25 Through 30	Test 30.25 30.30
66,000	25	35.36
65,616	25	35.36
52,500	15	26.72
42,500	27.5	27.5

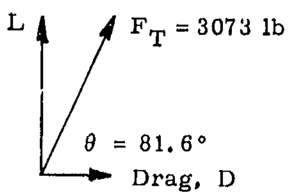
These increases at 52,500 ft and upward, Figure 21, represent a doubling of the dynamic pressure and resulting drag of the balloon in comparison to the original values at 65,616 ft. This change will also affect parts of the cable at these altitudes. The wind magnitudes below 52,500 ft and all azimuth values were left unchanged.*

In Table 5C, Test 30.25 uses the wind magnitudes shown above taken with the azimuths of Test 25 or two-dimensional case. In this test, the balloon moves from 63,316 down to 58,260 ft. In Test 30.30 the balloon level changes from 65,616 to 60,920 ft. In both cases the loss in altitude, between 4700 to 5000 ft, is due principally to balloon drag. Both of these tests indicate winch-tension increases of several hundred pounds coupled with additional decreases in cable elevation angle.

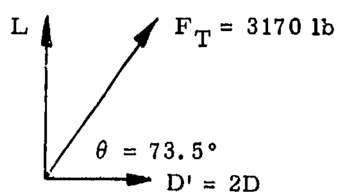
---

*In this exercise the balloon total force, F_T, and its angle, θ, were changed to 3170 lb and 73.5° by the following rationale. With the assumption of a natural-shape balloon having negligible aerodynamic lift, when the dynamic pressure is doubled, only the doubling of drag was considered. Changes of size and shape with altitude were considered negligible.

Orig. Condition



New Condition



It is evident that the increase in the balloon drag (estimated at approximately 450 lb) is felt at the winch. Changes in cable drag at the affected upper altitudes also add to the winch tension changes. Increases of more than 100 percent in the down range displacement may also be noted.

The results above indicate that the cable can be sized principally by considerations of basic balloon forces. The height, and cable density in addition will in most cases determine winch tension within reasonable working tolerances. Cable elevation angle at the winch becomes a good sensor of the collective effects of wind changes on the cable alone.

The test cases are shown here to illustrate the ease of using the programs to solve a typical tethered-balloon project's design and flight problems. A great many more points in a matrix of variables must be evaluated to cover all possible conditions that might be encountered during, for example, a flight of two-week's duration. Such a parametric study for the high-altitude tethered balloon would also have to include the effects of a changing balloon shape and size during ascent. Unlike the conventional tethered-balloon design having a ballonet, this balloon is a natural-shape type with factory installed reefing points for confining the excess material at altitudes below the maximum. As the balloon rises, one reefing point at a time is released to permit the gas to expand and maintain a non-flacid shape. Thus a series of values of balloon total-force F_T, and its angle, θ, must be evaluated for many altitudes to provide inputs for ascent studies.

PRECEDING PAGE BLANK - NOT FILMED

Appendix A

Symbols and Definitions

a_0, a_1	Constants used in Atmospheric Density Equation
A	Frontal Area of Cable Element, ft^2
AZ	Azimuth of the Wind, deg
AZ_B	Azimuth of the Balloon from the Winch Position, deg
AZ_C	Azimuth of the Cable Leaving the Winch, deg
C	Counter in 77.007B
C_D	Drag Coefficient of a Cylinder
D_T	Total Wind Drag on Cable Element, lb
D_H	Horizontal Component of D_T in Vertical Plane of Element, lb
D_S	Horizontal Component of D_T Perpendicular to Vertical Plane of Element, lb
D_W	Vertical Component of D_T, lb
F_T	Total Balloon Force, lb
g	Horizontal Projection of Element Length, K, ft
h	Horizontal Distance along Y_B axis, ft
H	Sum of Horizontal Distances, h, ft
i	Horizontal Distance along X_B axis, ft
I	Sum of Horizontal Distances, i, ft
j	Vertical Projection of Element Length, K, ft
J	Sum of Vertical Distances, j, or $Z_B - Z_S$, ft
K	Incremental Cable Element Length, ft

L	Horizontal Distance from Winch to Balloon, ft
M	Tension Tick-Mark Interval, lb (Plot)
N	Spacial Tick-Mark Interval, ft (Plot)
n_{CD}	Recall Code Number in C_D Computations
NW	Recall Code Number in Wind Computations
ON	Option Code Number
P	Rounding Factor (Plot)
q	Dynamic Pressure, $1/2\rho V^2$, lb/ft^2
r	Repeater Code Number in Optional Lower Altitude Runs in 77.007 and 77.007P or in 77.007B
R	Reynolds Number
T	Tension, lb
V	Wind Velocity, fps
V_A	Component of Wind Velocity In line with Element, fps
V_N	Component of Wind Velocity Normal to Element, fps
V_C	Horizontal Component of Wind Velocity in Vertical Plane of Element, fps
V_S	Horizontal Component of Wind Velocity Perpendicular to The Vertical Plane of the Element, fps
W	Weight of Cable Element, lb
Wd	Wind Velocity, knots
X_B	X-Axis Centered at Balloon, aligned 90° with Centerline of Balloon and Positive to Right of Balloon (View from Above)
X_W	X-Axis Centered at Winch, parallel with X_B axis but Positive Opposite to X_B
X	X-Axis Centered at Winch, aligned East-West, Positive East
Y_B	Y-Axis Centered at Balloon, aligned with Centerline of Balloon and Positive Forward of the Balloon
Y_W	Y-Axis Centered at Winch, parallel with Y_B axis, and Positive Opposite to Y_B
Y	Y-Axis Centered at Winch, aligned North-South, Positive North
Z	Altitude, ft MSL and Vertical Axis
Z_B	Balloon Altitude, ft MSL
Z_S	Surface Altitude, ft MSL

α	Relative Wind Angle to Vertical Element Plane, deg
β	Angle of Rotation between Adjacent Vertical Element Planes, deg
β_B	Azimuth of the Wind at the Balloon or the Azimuth to which the Balloon Points, deg
γ	Angle in the X-Y Plane between the X-Axis and the Straight Line from Winch to Balloon, deg
δ	Angle in the X-Y Plane between the X_W-Axis and the Straight Line from Winch to Balloon, deg
ϵ	Elevation Angle of Balloon from the Winch, deg
θ	Elevation of Element or Tension Vector above the Horizontal, deg
μ	Atmospheric Coefficient of Viscosity, lb/ft-sec
ρ	Atmospheric Density, slugs/ft^3 (For R, lb/ft^3)
ϕ	Angle between the V and V_N Vectors, deg
π	To Call Special Operation—Computation of C_D
Δ	Increment
Σ	Sum